

CHAPTER 11

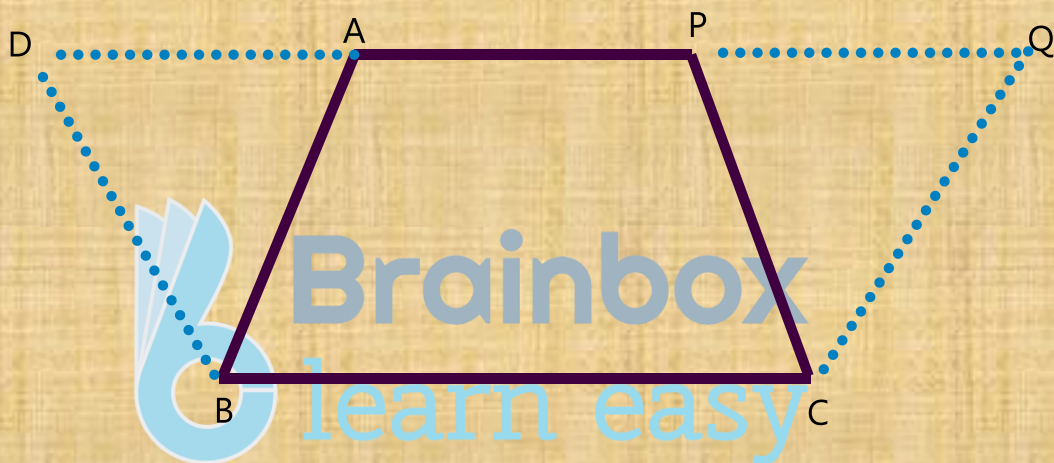
Areas

Triangles on the same base and between the same parallels:

Theorem 2:

Triangles on the same base and between the same parallels are equal in area.

Given:



Two triangles ABC and PBC, on the same base BC and between the same parallel lines BC and AP.

To prove:

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle PBC)$$

Construction:

Through B, draw $BD \parallel CP$, intersecting line AP in D and through C, draw $CQ \parallel BA$, intersecting line AP in Q.

Proof:

Quadrilateral BCQA is a parallelogram.

Quadrilateral BCPD is a parallelogram.

Parallelograms BCQA & BCPD are on the same base BC and between same parallels BC & AP.

$$\therefore \text{ar (Parallelogram BCQA)} = \text{ar (Parallelogram BCPD)} \dots\dots (1)$$

Now,

CA is a diagonal of parallelogram BCQA,

$$\therefore \text{ar}(\triangle ABC) = \text{ar}(\triangle AQC) = \frac{1}{2} \text{ar (Parallelogram BCQA)} \dots\dots (2)$$

$$\text{Similarly, ar}(\triangle PBC) = \frac{1}{2} \text{ar (Parallelogram BCPD)} \dots\dots (3)$$

From (1), (2) & (3),

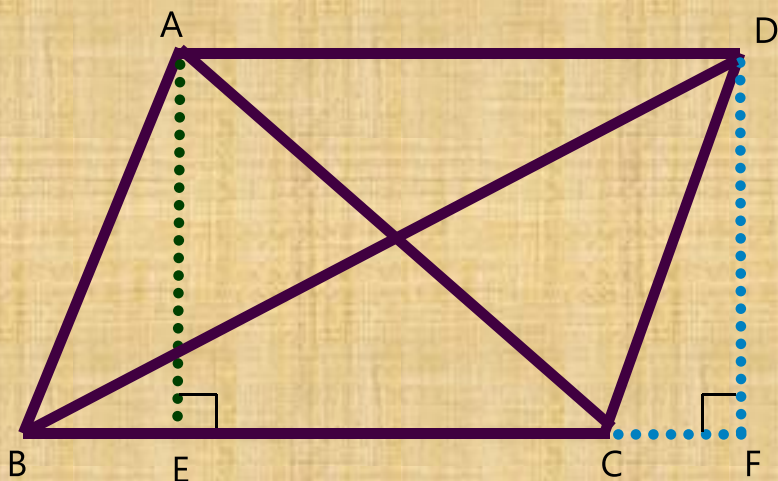
We get

$$\text{ar}(\triangle PBC) = \text{ar}(\triangle ABC)$$

Corollary 1:

Two triangles having the same base and equal areas will lie between the same parallels.

Proof:



Consider $\triangle ABC$ & $\triangle BCD$ such that

$\text{area}(\triangle ABC) = \text{area}(\triangle BCD)$ and they are on the same base BC.

Let AE, DF are the corresponding altitudes of two triangles.

We know that $\text{area}(\triangle ABC) = \frac{1}{2} \times BC \times AE$

(And)

$$\text{ar}(\triangle BCD) = \frac{1}{2} \times BC \times DF$$

But,

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle BCD)$$

$$\frac{1}{2} \times \cancel{BC} \times AE = \frac{1}{2} \times \cancel{BC} \times DF$$

$$\therefore AE = DF$$

i.e., A & D are equidistant from the side BC.

$\Rightarrow$ AD is parallel to BC.

$\Rightarrow \triangle ABC$ & $\triangle BCD$ lie between same parallels AD & BC.

