

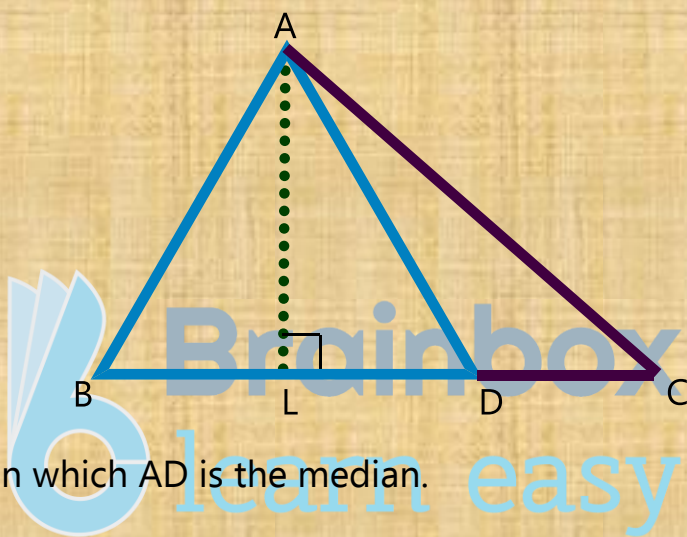
CHAPTER 11

Areas

Example:

Show that a median of a triangle divides it into two triangles of equal area.

Given:



$\triangle ABC$ in which AD is the median.

To prove:

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ADC)$$

Construction:

Draw $AL \perp BC$.

Proof:

Since 'AD' is the median of $\triangle ABC$.

Therefore, 'D' is the midpoint of BC.

$$\Rightarrow BD = DC$$

$$\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AL \dots\dots (1)$$

$$\text{ar}(\triangle ADC) = \frac{1}{2} \times DC \times AL \dots\dots (2)$$

But $BD = DC$

$\therefore$ From (1) & (2),

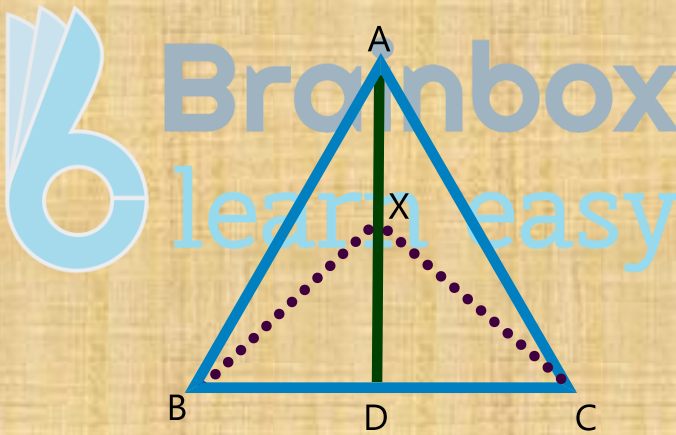
$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ADC)$$

Alternative:

Since $\triangle ABD$ and $\triangle ADC$ have equal bases and the same altitude AL .

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ADC)$$

Observation:



AD is the one of the medians of $\triangle ABC$.

'X' is any point on AD, then

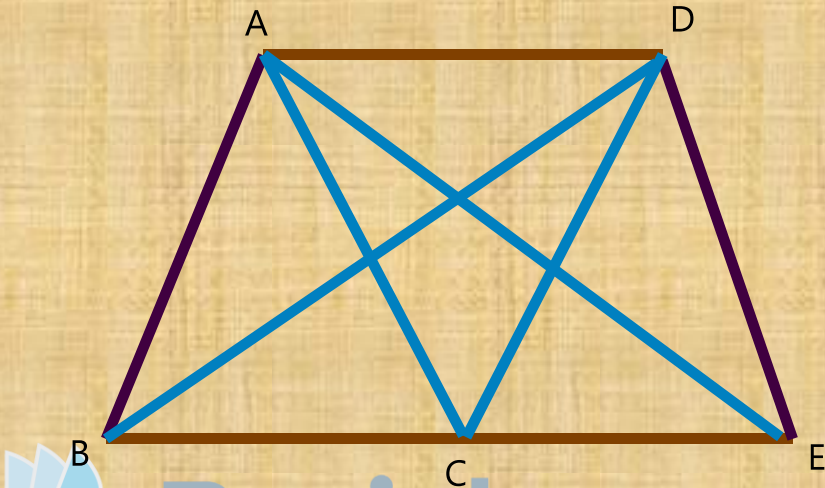
$$\text{ar}(\triangle ABX) = \text{ar}(\triangle ACX)$$

Example:

ABCD is a quadrilateral. AC is the diagonal and DE $\parallel$ AC and also DE meets BC produced at E.

Show that $\therefore \text{ar}(\text{ABCD}) = \text{ar}(\triangle \text{ABE})$.

Sol.



$$\text{ar}(\text{ABCD}) = \text{ar}(\triangle \text{ABC}) + \text{ar}(\triangle \text{DAC})$$

$\triangle \text{DAC}$ & $\triangle \text{EAC}$ lie on the same base AC and between same parallels, DE & AC.

$$\therefore \text{ar}(\triangle \text{DAC}) = \text{ar}(\triangle \text{EAC})$$

Adding areas of same figures on both sides.

$$\Rightarrow \text{ar}(\triangle \text{DAC}) + \text{ar}(\triangle \text{ABC}) = \text{ar}(\triangle \text{EAC}) + \text{ar}(\triangle \text{ABC})$$

$$\Rightarrow \text{ar}(\text{ABCD}) = \text{ar}(\triangle \text{ABE})$$