CHAPTER 16

Playing with Numbers

Reversing the digits 3 digit number:

Step I:

Let us consider a 3 digit number abc,

i.e. abc = 100a + 10b + c ………….(1)

Step II:

Reverse its digits, we get

cba = 100c + 10b + a ……….(2)

Step III:

Case (I):

If a > c,

Then difference between the numbers,

abc – cba = 100a + 10b + c – 100c –10b – a

= 99a – 99c

$\therefore \boxed{abc - cba = 99(a-c); a > c}$

Hence, the difference is divisible by 99 and quotient is a – c.

Case (II):

If c > a,

Then difference will become,

cba – abc = 100c + 10b + a – 100a – 10b – c

= 99c – 99a

$\therefore \boxed{cba - abc = 99(c - a); c > a}$

Hence, the difference is divisible by 99 and quotient is c – a.

Case (III):

If a = c, then difference is 0.

∴ In all cases, whether a > c or c > a or a = c, the difference is always divisible by 99 leaving remainder '0' and quotient is (a – c) or (c – a).

Note:

In general, if |a – c| = n, then the difference is 99n, [100(n – 1) + 10(9) + 9 (n – 1)]

Ex.

Without performing actual division, find the remainder and quotient when,

(i) Difference of 726 and 627 is divided by '99'.

(ii) Difference of 129 and 921 is divided by '8'.

Sol.

(i) 627 is obtained from 726 by reversing its digits, where 6 < 7.

∴ When difference is divided by 99 gives remainder zero and quotient is 7 – 6 = 1.

(ii) 921 is obtained from 129 by reversing its digits, where 9 > 1.

∴ Difference of numbers = 99 (9 – 1) = 99 x 8

Hence, when difference is divided by '8', it will give remainder zero and quotient 99.

(iii) Forming three digit numbers with given three digits:

Let the given 3 digit be abc,

By forming 3 digits are abc, bca, cab

Now,

abc + bca + ca + b = (100a + 10b + c) + (100a + 10c + a) + (100c + 10a + b)

= 111a + 111b + 111c

= 111(a + b + c)

= 37 x 3 (a + b + c) is always divisible by 37, 3 and sum of its digits.

Example:

Let us consider 3 digit number 623,

Sol.

Then 623 + 236 + 362 = 37 x 3 (6 + 2 + 3)

= 37 x 3 x 11

$\therefore$ Sum is divisible by 37, 3 and 11.

Note:

The 3 digits can be used to form six different 3 digit numbers abc, acb, bca, bac, cab and cba.