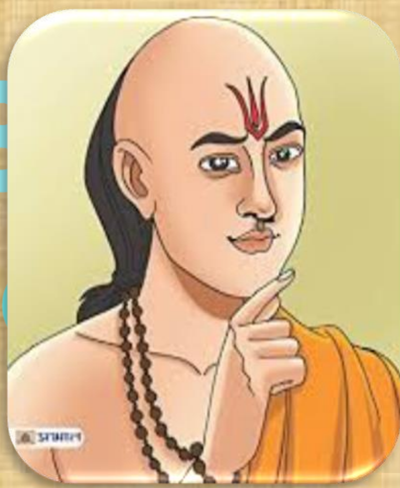
Chapter 02

POLYNOMIALS

Bhaskaracharya (1114-1185): Bhaskaracharya (also known as Bhaskara II) is regarded as one of the greatest Indian mathematician. He made significant contributions in mathematics and astronomy. He was born Bijjada Bida (in present day Bijapur district, Karnataka state). He was the head of the prestigious astronomical observatory at ujjain. His predecessors in this post were Brahmagupta and Varahamihira.



His Work:

He wrote siddhanta Shiromani in the year 1150 AD. It contained four sections.

a) Lilavati – dealing with arithmetic
b) Bijaganita – deals with algebra
c) Goladhyaya – deals with sphere and
d) Grahaganita – deals with mathematics of planets.

A number of his contributions were later taken to the Middle East and Europe.

ALGEBRA:

a) The recognition of a positive number having two square roots

b) Surds operations with products of several unknowns.

c) The solutions of:

- **i.** Quadratic equations
- **ii.** Cubic equations
- **iii.** Equations with more than one unknown.
- **iv.** Quadratic equations with more than one unknown.
- **v.** The general form of Pell's equation using the Chakravakam method.
- **vi.** The general indeterminate quadratic equation using the Chakravala method.
- **vii.** Indeterminate cubic equations.
- **viii.** Indeterminate quadratic equations.
- **ix.** Indeterminate higher-order Polynomial equations.

INTRODUCTION:

1. POLYNOMIALS:

An algebraic expression in which the variables involved only non-negative integral powers is called a polynomial.

(OR)

An expression of the form $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots\ldots + a_{n-1}x + a_n$.

Where $a_0, a_1, a_2, \ldots\ldots a_{n-1}, a_n$ are real numbers and $a_0 \neq 0$, is called a polynomial in $'x'$ of degree $'n'$.

Example:

1) $2x^2 - 5x + 4$ is a Polynomial.
2) $3x^4 - 7x^2 + \frac{5}{2}x + \frac{1}{3}$ is a Polynomial.

Value of a Polynomial $\mathbf{P(x)}$ at $\mathbf{x = \alpha}$:

The value of a Polynomial $\mathbf{P(x)}$ at $\mathbf{x = \alpha}$ is obtained by substituting $\mathbf{x = \alpha}$ in the given polynomial and it is denoted by $\mathbf{P(\alpha)}$.

Example: If $P(x) = 2x^3 + 3x^2 - 11x - 6$ find $P(-1)$.

Sol: Given,

$P(x) = 2x^3 + 3x^2 - 11x - 6$

Put $x = -1$

$$P(-1) = 2(-1)^3 + 3(-1)^2 - 11(-1) - 6$$
$$= 2(-1) + 3(1) + 11 - 6$$
$$= -2 + 3 + 11 - 6$$
$$\therefore \boxed{P(-1) = 6}$$

2. Degree of Polynomial in one Variable:

In a Polynomial in one variable, the highest power of the variable is called its **"degree"**.

Example:

1) $3x^{\boxed{1}} + 4$ is a polynomial in 'x' of **degree 1.**

2) $3x^{\boxed{4}} - 7x^2 + \frac{5}{2}x + \frac{1}{3}$ is a polynomial in $'x'$ of **degree 4.**

3. Degree of a Polynomial in two variables:

In a Polynomial of more than one variable, the sum of the powers of variables in each term is computed and the **highest sum** so obtained is called the degree of the Polynomial.

Example:

1) $3x^4 - 2x^3y^2 + 7xy^3 - 9x + 5y + 4$ is Polynomial in $'x'$ and $'y'$ of degree 5.

Sol: Degree of $x^3y^2 = 3 + 2 = 5$.

4. Constant Polynomial:

A Polynomial consisting of a constant term only is called a **"Constant Polynomial".**

<u>**NOTE**</u>**:**

- The degree of a constant Polynomial is **"zero".**

Example: 5, 7.

5. Linear Polynomial:

A Polynomial of degree '1' is called a **"Linear Polynomial".**

Example:

1) $\frac{1}{2}x + \frac{3}{5}$

2) $3 - \frac{5}{7}y$

6. Quadratic Polynomial:

A Polynomial of degree '2' is called a **"Quadratic Polynomial".**

Example:

1) $5y^2 + 4y$

2) $2 - x + x^2$

7. Cubic Polynomial:

A Polynomial of degree '3' is called a **"Cubic Polynomial".**

Example:

1) $2x^3 + 4x^2 + 6x + 7$

2) $x^3 - 7x^2 + 2x - 3$

8. Bi quadratic Polynomial:

A Polynomial of degree '4' is called a **"Bi quadratic Polynomial".**

Example:

1) $x^4 + 2x^3 - 2x - 1$

2) $3x^4 - 7x^3 + x^2 - x + 9$

9.

Degree	Polynomial	Examples
1	Linear	$\frac{1}{2}x^{\boxed{1}}+\frac{3}{5}$
2	Quadratic	$5y^{\boxed{2}}+4y$
3	Cubic	$2z^{\boxed{3}}+4z^2+6z+7$
4	Biquadratic	$t^{\boxed{4}}+2t^3-2t-1$

NOTE:

- The degree of zero Polynomial is "not defined".
- Every Polynomial is an algebraic expression but an algebraic expression need not be a Polynomial.

Classification of polynomials based on number of terms:

- A Polynomial contains one term is called **monomial.**
- A Polynomial contains two terms is called **Binomial**.
- A Polynomial contains three terms is called **Trinomial**.

10. Zero's of a Polynomial:

If $f(x)$ is a Polynomial in one variable then $f(k)$ is zero then 'k' is called as 'zero' of the Polynomial $f(x)$.

Example:

$f(x)=x^3-2x^2-x+2$. When $x=1$ the value of $f(x)$ is zero.

$$f(1)=(1)^3-2(1)^2-1+2$$
$$=\cancel{1}-\cancel{2}-\cancel{1}+\cancel{2}$$
$$=0$$

$\therefore$ '1' is a called a 'zero' of Polynomial $f(x)$.

NOTE:

- 'Zero' of a Polynomial is also called as root.

11. Difference between Multinomial and Polynomial:

a) A Polynomial is concerned with positive integral power of the variable involved in the expression and not with the number of terms in the expression.

b) A Multinomial is concerned with number of terms in the expression and not with the powers of the variable in the expression.

Examples:

1) $x^3 - 2x^2 - x + 2$ is a Polynomial.

2) $3\sqrt{x} - \frac{4}{x}$ is a Multinomial.

<u>NOTE</u>:

- All Polynomials are multinomial. But all multinomials are not Polynomials.

12. Difference between zero Polynomial and zero of the Polynomial:

Zero Polynomial is one type of polynomial whereas zero of a polynomial is a number at which the value of the polynomial is zero.

13. Division of a polynomial by another polynomial:

Example: Divide $3x^2 + x^3 + 3x + 1$ by $x + 1$.

Sol:

Step-1:

Arrange the terms of the dividend and divisor in descending (ascending) order of their degrees.

i.e., $x^3 + 3x^2 + 3x + 1$ and $x + 1$.

$$\underset{\text{(Divisor)}}{x+1}\,\Big)\,\underset{\text{(Dividend)}}{x^3+3x^2+3x+1}\,\Big(\,\underset{\text{(Quotient)}}{?}$$

Step-2:

We get first term of quotient by dividing the first term of dividend with the first term of divisor. i.e., $\frac{x^3}{x} = x^2$, So first term of quotient = x^2

$$\begin{array}{l} x+1)\,x^3+3x^2+3x+1\,(x^2 \\ \qquad\; x^3+x^2 \\ \hline \end{array}$$

Step-3:

Multiply the divisor by the first term of the quotient and subtract the result from the dividend to get remainder.

i.e.

$$\begin{array}{l} x+1)\,\cancel{x^3}+3x^2+3x+1\,(x^2 \\ \qquad\; \underset{(-)}{\cancel{x^3}} \underset{(-)}{+} x^2 \quad \downarrow \quad \downarrow \\ \hline \qquad\qquad 2x^2+3x+1 \rightarrow \text{Remainder} \end{array}$$

Step-4:

Do the same as in Step-2 to get second term of quotient by taking the remainder in Step-3 as dividend.

i.e.,

$$
\begin{array}{l}
x+1)\,\cancel{x^3}+3x^2+3x+1\,(x^2+2x+1 \\
\underline{\;{}_{(-)}\cancel{x^3}+_{(-)}x^2\;} \\
\qquad \cancel{2x^2}+3x+1 \\
\qquad \underline{{}_{(-)}\cancel{2x^2}+_{(-)}2x\downarrow} \\
\qquad\qquad \cancel{x}+\cancel{1} \\
\qquad\qquad \underline{{}_{(-)}\cancel{x}+_{(-)}\cancel{1}} \\
\qquad\qquad\;\; 0 \;\rightarrow \text{Remainder}
\end{array}
$$

Again step-2: $\frac{\cancel{x}}{\cancel{x}}=1$

By Step-2: $\frac{2x^2}{2}=\frac{2x.\cancel{x}}{\cancel{x}}=2x$, By Step-3: Subtract.

Step-5:

Repeat the above process till we obtain a remainder that is either zero or a Polynomial of degree less than that of the divisor. i.e., **<u>0</u>.**

14. Division Principle:

Dividend = (Divisor x Quotient) + Remainder

Where remainder is '0' or a polynomial of degree less than that of divisor.

Some Notations:

i) Dividend is $\mathrm{P}(x), \mathrm{F}(x), p(x)\,(\text{or})\, f(x)$ etc.;

ii) Divisor is $g(x)$

iii) Quotient is $q(x)$

iv) Remainder is R or $r(x)$

v) If divisor is first degree than remainder is of '0' degree i.e., a constant.

Hence 'R' is the remainder if $(x-a)(x-b),(ax+b)$ or $(ax-b)$ are the divisors.

15. Remainder theorem:

If $f(x)$ is a polynomial then the remainder of $f(x)$ when divided by $(x-a)$ is $f(a)$.

<u>NOTE</u>:

- Reminder is usually obtained at the end of division process. But using division principle, remainder is obtained without direct division.

  i.e., $f(x)=g(x).q(x)+R \rightarrow (1)$

  Let $g(x)=x-a$

  From (1)

  $\Rightarrow f(x)=(x-a)q(x)+R$

  Substitute $x=a$

  $\Rightarrow f(a)=0.q(a)+R \;\Rightarrow f(a)=R$

- Divisor and their respective remainders are presented in a table below.

SI NO	Divisors	Remainders
1	$(x-a)$	$f(a)$
2	$(x+a)$	$f(-a)$
3	$(ax+b)$	$f\left(-\frac{b}{a}\right)$
4	$(ax-b)$	$f\left(\frac{b}{a}\right)$

- Remainder theorem is having a limitation that it gives us the remainder only for first-degree divisors.
- Remainder when any first-degree divisor divides the Polynomial $f(x)$ is a constant.

Ex: Divide $f(x)=3x^2+x^3+3x+1$ by $x+1$.

Sol:

Dividend = x^3+3x^2+3x+1

Divisor = $g(x)=x+1$

$$\underset{\text{Divisor}}{x+1}\overset{\text{Quotient}}{\overline{\Big)\underset{\text{(Divident)}}{x^3+3x^2+3x+1}}}$$

$g(x)=0$

$\Rightarrow x+1=0$

$\therefore x=-1$

Substitute $x=-1$ in $f(x)$

$f(-1)=3(-1)^2+(-1)^3+3(-1)+1$

$=3(1)-1-3+1$

$=\cancel{3}-\cancel{1}-\cancel{3}+\cancel{1}$

$f(-1)=0$

Which is in the form of $f(a)=R$

$\therefore \boxed{\text{Remainder} = 0}$

16. Factor Theorem:

If $f(x)$ be a Polynomial and 'a' be a real number. Then $(x-a)$ is a factor of $f(x) \Leftrightarrow f(x)=0$.

Ex: Check $(x-2)$ is a factor of $x^3+3x^2-4x-12$

Sol: Let $g(x)=x-2=0$

$\Rightarrow \boxed{x=2}$

Let $f(x)=x^3+3x^2-4x-12$

If $x=2$

$f(2)=(2)^3+3(2)^2-4(2)-12$

$=\cancel{8}+\cancel{12}-\cancel{8}-\cancel{12}$

$f(2)=0$

$\therefore$ $(x-2)$ is a factor of $x^3+3x^2-4x-12$.

NOTE:

- $f(x)$ is a Polynomial then $(x-1)$ is a factor of $f(x)$ if the sum of Co-efficient of powers of
  'x' in $f(x)$ is zero.
- $f(x)$ is a Polynomial then $(x+1)$ is a factor of $f(x)$ if the sum of Co-efficients of even powers of 'x' is equal to the sum of the Co-efficients of odd powers of 'x' in $f(x)$.

17. Fundamental theorem of algebra:

Every Polynomial function of degree ≥ 1 has at least one zero in the complex number (Real or imaginary).

NOTE:

- Every Polynomial of degree $n \geq 1$ has exactly 'n' zeros.

18. Some Important Results:

(I) $x^n - y^n$ is divisible by $x-y$ for every positive integer 'n'.

Example:

a) $\dfrac{x^2-y^2}{x-y}=x+y$

b) $\dfrac{x^3-y^3}{x-y}=x^2+xy+y^2$

c) $\dfrac{x^4-y^4}{x-y}=x^3+x^2y+xy^2+y^3$

NOTE: 1

i. The divisor being '$x-y$'.

ii. The terms in the quotient all positive.

iii. The index in the dividend either odd or even.

NOTE: 2

If 'n' is odd (or) even.

$$\frac{x^n - y^n}{x - y} = x^{n-1} + x^{n-2}.y + x^{n-3}.y^2 + ...+ xy^{n-2} + y^{n-1}.$$

(II) $x^n - y^n$ is divisible by $x + y$ for every even positive integer 'n'.

Example:

a) $\frac{x^2 - y^2}{x + y} = x - y$

b) $\frac{x^4 - y^4}{x + y} = x^3 - x^2y + xy^2 - y^3$

c) $\frac{x^6 - y^6}{x + y} = x^5 - x^4y + x^3y^3 - x^2y^3 + xy^4 - y^5$

NOTE: 1

i. The divisor being $x + y$.

ii. The term in the quotient alternately positive and negative.

iii. The index in the dividend always even.

NOTE: 2

If 'n' is even.

$$\frac{x^n - y^n}{x + y} = x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 + ...+ xy^{n-2} - y^{n-1}.$$

(III) $x^n + y^n$ is divisible by $x + y$ for every odd positive integer 'n'.

Example:

a) $\frac{x^3 + y^3}{x + y} = x^2 - y.x + y^2$

b) $\frac{x^5 + y^5}{x + y} = x^4 - x^3y + x^2y^2 - xy^3 + y^4$

c) $\frac{x^7 + y^7}{x + y} = x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6$

NOTE: 1

i. The divisor being $x + y$

ii. The terms in the quotient alternately positive and negative.

iii. The index in the dividend always odd.

NOTE: 2

If 'n' is odd.

$$\frac{x^n + y^n}{x + y} = x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - ....- xy^{n-2} + y^{n-1}.$$

19. Identities:

Identity:

An identity is a statement that two expressions are equal for all values of the letters involved.

Example:

$7x = 2x + 5x$ is an identity.

NOTE:

i. Here the expression $7x$ and $2x + 5x$ are equal for all values of $'x'$.

ii. The sides of this identity are $7x$ and $2x + 5x$, $7x$ being the left hand side and $2x + 5x$ being the right hand side.

20. Algebraic Identities:

i) $(a+b)^2 = a^2 + 2ab + b^2$

ii) $(a-b)^2 = a^2 - 2ab + b^2$

iii) $(a+b)(a-b) = a^2 - b^2$

iv) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (or) $(a+b)^3 = a^3 + 3ab(a+b) + b^3$

v) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ (or) $(a-b)^3 = a^3 - 3ab(a-b) - b^3$

vi) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

vii) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

viii) $(a+b)(c+d) = ac + ad + bc + bd$

ix) $(x+a)(x+b) = x^2 + x(a+b) + ab$

x) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

xi) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

xii) $(a+b)^2 - (a-b)^2 = 4ab$

xiii) $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

xiv) $a^k + b^k = \left(a^{k-1} + b^{k-1}\right)(a+b) - \left(a^{k-2} + b^{k-2}\right)ab$

xv) $a^3 + b^3 + c^3 - 3abc = (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

Special Identities:

1) If $a+b+c=0$ then

- $a^2 + b^2 + c^2 = -2(ab + bc + ca)$
- $a^3 + b^3 + c^3 = 3abc$
- $a^4 + b^4 + c^4 = 2\left(a^2b^2 + b^2c^2 + c^2a^2\right) = \frac{1}{2}\left(a^2 + b^2 + c^2\right)^2$
- $a^5 + b^5 + c^5 = -5abc(ab + bc + ca) = \frac{5}{2}abc\left(a^2 + b^2 + c^2\right) = \frac{5}{6}\left(a^2 + b^2 + c^2\right)\left(a^3 + b^3 + c^3\right)$
- $a^7 + b^7 + c^7 = 7abc(ab + bc + ca)^2 = \frac{7}{12}\left(a^2 + b^2 + c^2\right)\left(a^3 + b^3 + c^3\right) = \frac{7}{6}\left(a^4 + b^4 + c^4\right)\left(a^3 + b^3 + c^3\right)$

2) If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}$ then $\frac{1}{a^{2n+1}} + \frac{1}{b^{2n+1}} + \frac{1}{c^{2n+1}} = \frac{1}{(a+b+c)^{2n+1}}$.

21. Use of Factor Theorem in Factorization:

By Trial & Error Method For Finding Linear Factors Of $f(x)$:

Let $f(x)$ be the given polynomial

i) Let the constant term in $f(x)$ be 3 (or) -3.

ii) Then its factors are 1, -1, 3 and -3.

iii) Find $f(1)$, $f(-1)$, $f(3)$ and $f(-3)$.

See which one is zero.

If $f(1) = 0$, then $(x-1)$ is a factors of $f(x)$.

If $f(-1)=0$, then $(x+1)$ is a factor of $f(x)$.

Hence by taking the factors of the constant term in the given polynomial. We can find a linear factor of $f(x)$ by trial and error method.

SOLVED PROBLEMS

1. Find the remainder when $\left(2x^3-3x^2+7x-8\right)$ is divided by $(x-1)$.

Sol: Let, $\mathrm{P}(x)=2x^3-3x^2+7x-8$

By Remainder theorem,
On dividing $\mathrm{P}(x)$ by $(x-1)$, we get

Remainder = $\mathrm{P}(1)$

$\Rightarrow \mathrm{P}(1)=2(1)^3-3(1)^2+7(1)-8=2-3+7-8=-2$

Hence, the required remainder is '-2'.

2. If $f(x)=\left(2x^3+x^2+bx-6\right)$ leaves a remainder 36 when divided by $(x-3)$, find the value of 'b'.

Sol: Given, $f(x)=2x^3+x^2+bx-6 \rightarrow (1)$

$\because$ $f(x)$ divided by $(x-3)$ leaves a remainder 36.

$\Rightarrow f(3)=36$

$\Rightarrow 2(3)^3+(3)^2+b(3)-6=36$ $\qquad$ ($\because$ From (1))

$\Rightarrow 2(27)+9+3b-6=36$

$\Rightarrow 3b+57=36$

$\Rightarrow 3b=36+57=-21$

$$b=\frac{\cancel{-21}^{\,7}}{\cancel{3}}$$

$\therefore \boxed{b=-7}$

3. Show that $(x-1)$ is a factor of $x^3-7x^2+14x-8$. Hence, completely factorize the above expression.

Sol: Let $\mathrm{P}(x)=x^3-7x^2+14x-8$

By factor theorem,

$(x-1)$ Will be a factor of $\mathrm{P}(x)$, if $\mathrm{P}(1)=0$

$\Rightarrow \mathrm{P}(1)=(1)^3-7(1)^2+14(1)-8$

$=\cancel{1}-\cancel{7}+\cancel{14}-\cancel{8}$

$\mathrm{P}(1)=0$

$\therefore (x-1)$ is a factor of $\mathrm{P}(x)$.

On dividing $\mathrm{P}(x)=x^3-7x^2+14x-8$ by $(x-1)$

We get,

Quotient = x^2-6x+8

$$
\begin{array}{r|l}
 & x^2-6x+8 \\
x-1 & \cancel{x^3}-7x^2+14x-8 \\
 & \cancel{x^3}-x^2 \\
 & {\scriptstyle (-)\quad (+)} \\
\hline
 & \cancel{-6x^2}+14x-8 \\
 & \cancel{-6x^2}+6x \\
 & {\scriptstyle (+)\quad (-)} \\
\hline
 & \cancel{8x-8} \\
 & \cancel{8x-8} \\
\hline
 & 0 \\
\hline
\end{array}
$$

$\therefore \mathrm{P}(x)=x^3-7x^2+14x-8$

$=(x-1)(x^2-6x+8)$

$=(x-1)(x^2-4x-2x+8)$

$=(x-1)[x(x-4)-2(x-4)]$

$=(x-1)(x-4)(x-2)$

Hence, $x^3-7x^2+14x-8=(x-1)(x-4)(x-2)$.

4. Find the values of the constants 'a' and 'b', if $(x-2)$ and $(x+3)$ are both factors of the expression $(x^3+ax^2+bx-12)$.

Sol: Let $P(x)=x^3+ax^2+bx-12$

Q $(x-2)$ is a factor of $P(x)$

$\Rightarrow P(2)=0$

$\Rightarrow (2)^3+a(2)^2+b(2)-12=0$

$\Rightarrow (-3)^2+a(-3)^2+b(-3)-12=0$

$\Rightarrow 4a+2b=4$

$\Rightarrow 2a+b=2 \rightarrow (1)$

Q $(x+3)$ is a factor of $P(x)$

$\Rightarrow P(-3)=0$

$\Rightarrow 9a-3b=39$

$\Rightarrow 3a-b=13 \rightarrow (2)$

Add (1) & (2) we get,

$5a=15$

$\Rightarrow a=\dfrac{\cancel{15}}{\cancel{5}}=3$

$\Rightarrow \boxed{a=3}$

Substitute 'a' in equation (1),

$2(3)+b=2$

$\Rightarrow 6+b=2$

$\Rightarrow b=2-6=-4$

$\Rightarrow \boxed{b=-4}$

Hence, $\boxed{a=3}$ and $\boxed{b=-4}$.

5. Find the value of $\dfrac{a}{(a-b)(a-c)}+\dfrac{b}{(b-c)(b-a)}+\dfrac{c}{(c-a)(c-b)}$.

Sol: $\dfrac{-a(b-c)-b(c-a)-c(a-b)}{(a-b)(b-c)(c-a)}=\dfrac{0}{(a-b)(b-c)(c-a)}=0$

6. Use the factor theorem to factorize completely $\left(x^3+x^2-4x-4\right)$.

Sol: Let $f(x)=x^3+x^2-4x-4$

The constant term in $f(x)$ is '-4'

Its factors are 1, -1, 2, -2, and -4.

By trial and error method,

$f(2)=2^3+2^2-4(2)-4=0$

$\therefore (x-2)$ is a factor of $f(x)=x^3+x^2-4x-4$

On dividing $f(x)$ by, $(x-2)$ we get,

Quotient = $\left(x^2+3x+2\right)$

$\therefore x^3+x^2-4x-4=(x-2)\left(x^2+3x+2\right)$

$=(x-2)\left(x^2+2x+x+2\right)$

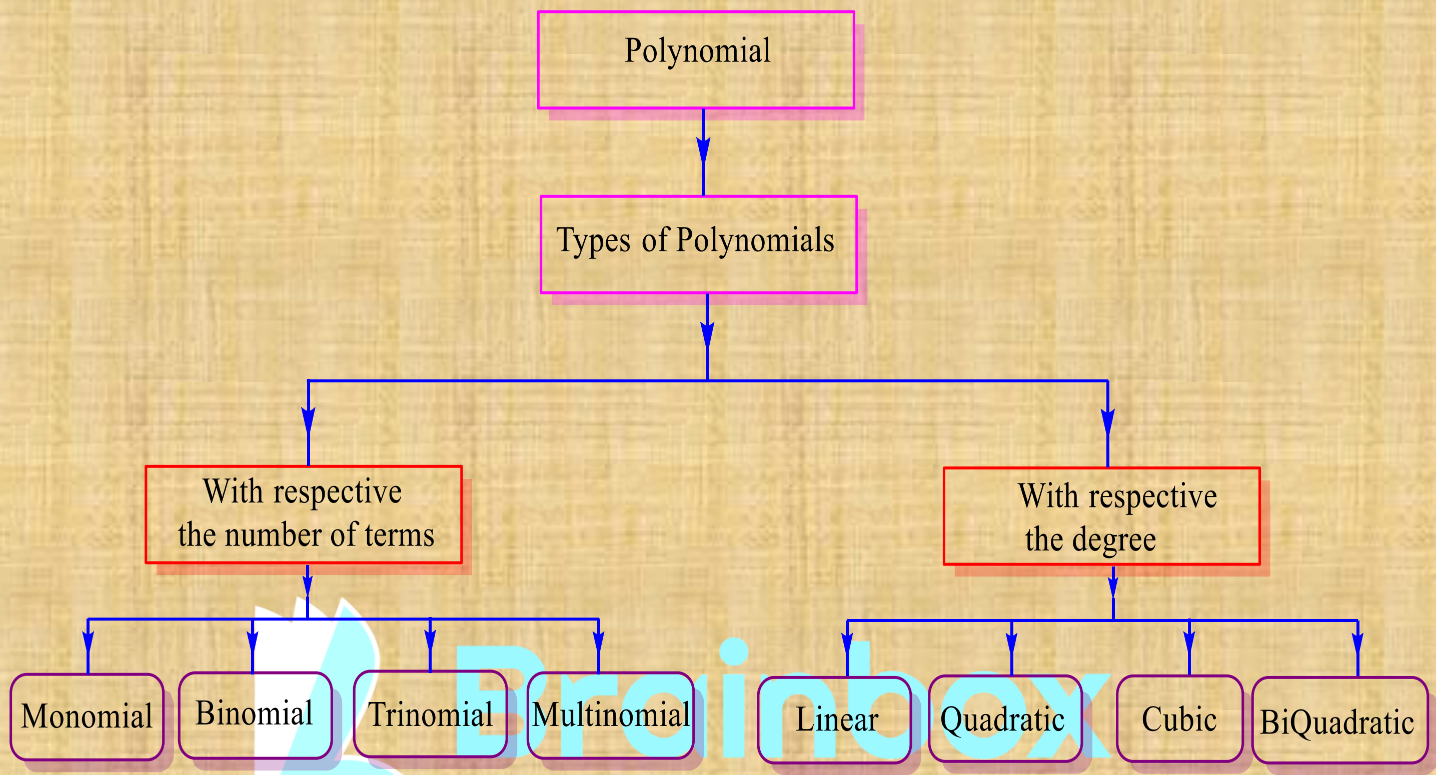
$=(x-2)\left(x(x+2)+1(x+2)\right)$

$=(x-2)\left((x+2)(x+1)\right)$

Hence, $x^3+x^2-4x-4=(x+1)(x-2)(x+2)$

$$
\begin{array}{r|l}
 & x^2+3x+2 \\
\hline
x-2 & \cancel{x^3}+x^2-4x-4 \\
 & \cancel{x^3}-2x^2 \\
 & (-)\quad(+) \\
\hline
 & \cancel{3x^2}-4x-4 \\
 & \cancel{3x^2}-6x \\
 & (-)\quad(+) \\
\hline
 & \cancel{2x-4} \\
 & \cancel{2x-4} \\
\hline
 & 0
\end{array}
$$

CONCEPT MAP



*** THE END ***