Chapter 02

POLYNOMIALS

Example:

Show that $(x-1)$ is a factor of $x^3-7x^2+14x-8$. Hence, completely factorise the above expression.

Solution:

Let $\mathrm{P}(x)=x^3-7x^2+14x-8$

By factor theorem,

$(x-1)$ will be a factor of $\mathrm{P}(x)$, if $\mathrm{P}(1)=0$

$$\Rightarrow \mathrm{P}(1)=(1)^3-7(1)^2+14(1)-8$$

$$=\cancel{1}-\cancel{7}+\cancel{14}-\cancel{8}$$

$$\mathrm{P}(1)=0$$

$\therefore (x-1)$ is a factor of $\mathrm{P}(x)$.

On dividing $\mathrm{P}(x)=x^3-7x^2+14x-8$ by $(x-1)$

$$\begin{array}{r|l} & x^2-6x+8 \\ \hline x-1 & \cancel{x^3}-7x^2+14x-8 \\ & \underset{(-)}{\cancel{x^3}}\ \underset{(+)}{-x^2} \\ \hline & \cancel{-6x^2}+14x-8 \\ & \underset{(+)}{\cancel{-6x^2}}\ \underset{(-)}{+6x} \\ \hline & \cancel{8x-8} \\ & \cancel{8x-8} \\ \hline & 0 \end{array}$$

We get, Quotient = x^2-6x+8

$$\therefore \mathrm{P}(x)=x^3-7x^2+14x-8$$

$$=(x-1)(x^2-6x+8)$$

$$=(x-1)(x^2-4x-2x+8)$$

$$=(x-1)[x(x-4)-2(x-4)]$$

$$=(x-1)(x-4)(x-2)$$

Hence, $x^3-7x^2+14x-8=(x-1)(x-4)(x-2)$.