CHAPTER 09

Areas of Parallelogram and Triangles

Relationship between the parallelograms (Having equal areas):

Theorem 1:

Parallelograms on the same base and between the same parallel lines are equal in area.

Given:

Two parallelograms ABCD and ABEF are on the same base AB and between the same parallel lines AB and FC as shown in figure.

To prove that,

Area [Parallelogram ABCD] = Area [Parallelogram ABEF]

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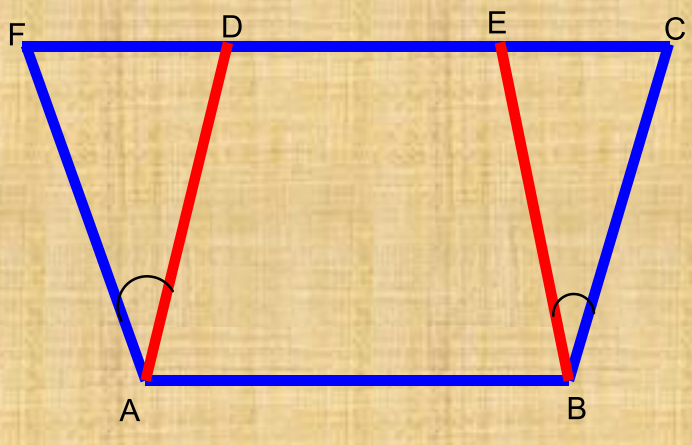
Proof:

In ΔADF and ΔBCE, we have

AD = BC [Opposite sides of a parallelogram ABCD]

AF = BE [Opposite sides of parallelogram ABEF]



$\angle DAF = \angle CBE$

[Why because AD || BC and AF || BE. So angle between sides AD and AF is equal to angle between sides BC and BE]

$\therefore \Delta ADF \cong \Delta BCE$ (By SAS congruency rule)

Area (ΔADF) = Area (ΔBCE), which means congruent figures have equal areas.

Now, Area (Parallelogram ABCD) = Area (ABED) + Area (ΔBCE)

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Which is equal to Area (Parallelogram ABEF)

Hence, Area (Parallelogram ABCD) = Area (Parallelogram ABEF)

Parallelograms ABCD and ABEF are equal in area.

From the above theorem, we can prove that,

- A parallelogram and a rectangle on the same base and between the same parallels are equal in area since a rectangle is also a parallelogram.
- Parallelograms on equal bases and between the same parallels are equal.
- Area of parallelogram = Base x Height

We have two triangles ABC and PBC on the same base BC and AP.

For areas of such triangles,

Draw several pairs of triangles on the same base and between the same parallels on the graph sheet and find their areas by the method of counting the squares.

Each time, you will find that the areas of the two triangles are approximately equal.

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Theorem 2:

Statement:

If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

Given:

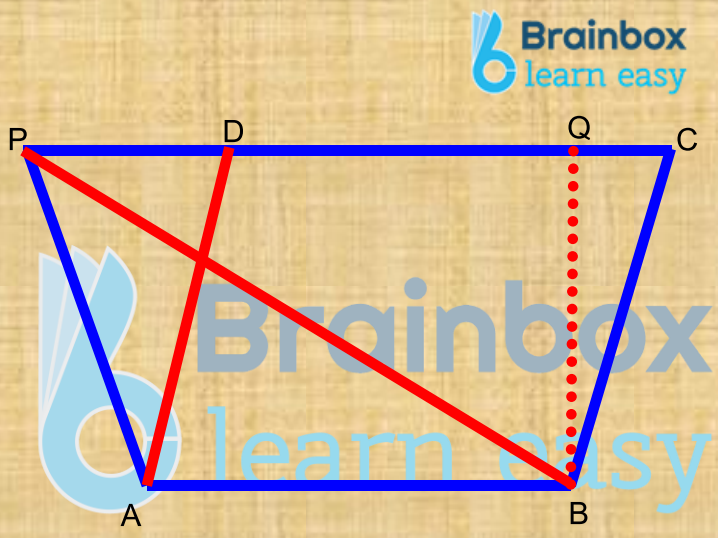
ΔABP and Parallelogram ABCD are on the same base AB and between the same parallels AB and PC.

To prove that Area $(\Delta PAB) = \frac{1}{2}$ Area (Parallelogram ABCD)

Draw BQ || AP to obtain another parallelogram ABQP.

Proof:

Parallelograms ABQP and ABCD are on the same base AB and between the same parallels AB and PC.



Area (Parallelogram ABQP) = Area (Parallelogram ABCD)…….(1)

Area $(\Delta PAB) \cong$ Area (ΔBQP)

Because diagonal PB divides parallelogram ABQP into two congruent triangles.

Area (ΔPAB) = Area (ΔBQP) …… (2)

From equations (1) and (2), we can say that,

$$\text{Area } (\Delta PAB) = \frac{1}{2} \text{ Area (Parallelogram ABPQ)}$$

$$\text{Area } (\Delta PAB) = \frac{1}{2} \text{ Area (Parallelogram ABCD)}$$

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