CHAPTER 09

Areas of Parallelogram and Triangles

Theorem 3:

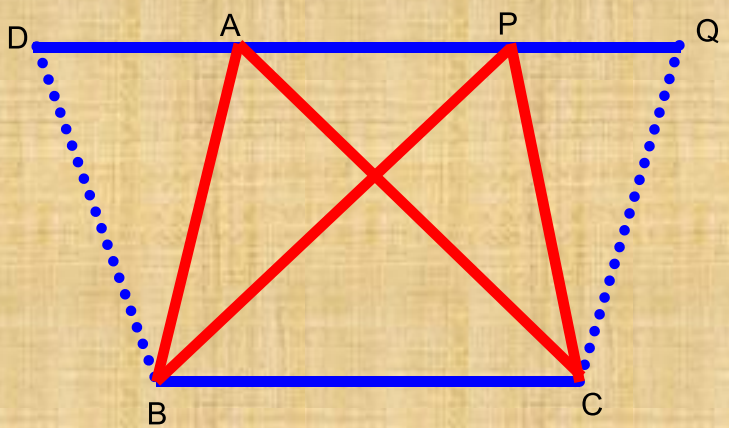
Statement:

Triangles on the same base and between the same parallels are equal in area.

Given:

Two triangles ABC and PBC are on the same base BC and between the same parallel lines BC and AP.

To prove that Area (ΔABC) = Area (ΔPBC)



Construction:

Through B, draw BD || CP intersecting line AP at D and through C. Draw CQ || BA intersecting line AP at Q.

Proof:

Quadrilaterals BCQA and BCPD are parallelograms. We can observe that, parallelograms BCQA and BCPD are on the same base BC and between the same parallels BC and DQ.

Area (Parallelogram BCQA) = Area (Parallelogram BCPD).....(1)

Now, CA is a diagonal of parallelogram BCQA.

$\therefore$ Area (ΔABC) = Area (ΔAQC) = $\frac{1}{2}$ Area (Parallelogram BCQA)

..... (2)

Similarly,

Area (ΔPBC) = $\frac{1}{2}$ Area (Parallelogram BCPD)..... (3)

From (1), (2) & (3)

We get Area (ΔPBC) = Area (ΔABC)

Theorem 4:

Statement:

The area of a triangle is half the product of any its sides and corresponding altitude.

Given that:

ΔABC in which AL is the altitude to the side BC.

To prove that Area $(\Delta ABC) = \frac{1}{2}(BC) \times AL$

Construction:

Through 'C' and 'A' draw lines parallel to BA and BC respectively intersecting each other at D.

Proof:

Quadrilateral ABCD is a parallelogram [By construction]

$\because$ AC is a diagonal of parallelogram ABCD.

$\therefore$ Area $(\Delta ABC) = \frac{1}{2}$ Area (Parallelogram ABCD)

[In theorem 2]……..(1)

BC is a side of parallelogram BCDA and AL is the corresponding altitude.

$\therefore$ Area (Parallelogram ABCD) = BC x AL…….(2)

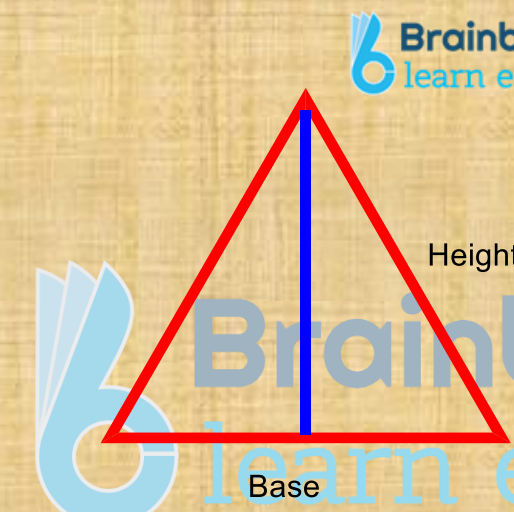
From (1) & (2)

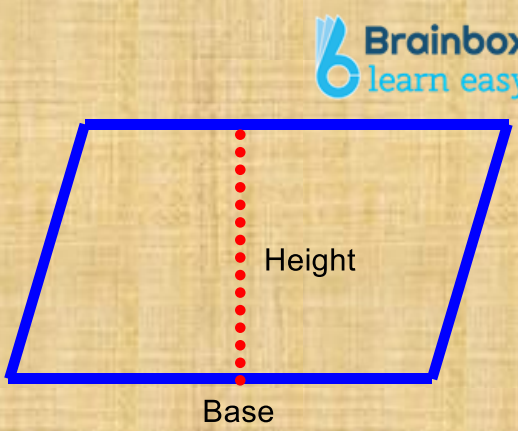
We get Area $(\Delta ABC) = \frac{1}{2}(BC) \times AL$

Let us recall the formulae,

Area of triangle = $\frac{1}{2}$ x Base x Triangle

Area of parallelogram = Base x Height





Area of rectangle = Length x Breadth

Area of trapezium = $\frac{1}{2}$ x (Sum of parallel sides) x (Distance between them)

$= \frac{1}{2} \times (a + b) \times h$

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

[Where d_1 = Length of one diagonal,

d_2 = Length of another diagonal]