CHAPTER 01

Number System

Laws of exponents for real numbers:

Let a, b (>0) be real numbers and m, n be two rational numbers, then the following laws hold good for rational exponents.

(i) $a^m \times a^n = a^{m+n}$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iii) $\left(a^m\right)^n = a^{mn}$

(iv) $a^{-n} = \dfrac{1}{a^n}$

(v) $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$

(vi) $(ab)^m = a^m b^m$

(vii) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

(viii) $a^{\frac{m}{n}} = \left(a^m\right)^{1/n} = \left(x^{1/n}\right)^m$ i.e. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

(ix) $a^{b^n} = a^{b \times b \times b \ldots (n\ times)}$

Note:

For every non – zero real number a, $a^0 = 1$.

Example:

Simplify,

(i) $(128)^{1/7}$ **(ii)** $7^{1/17}.11^{1/17}$ **(iii)** $\frac{5^2}{5^3}$ **(iv)** $(0.00032)^{\frac{-2}{5}}$

Sol.

(i) $(128)^{1/7} = \left(2^7\right)^{1/7} = 2^{7\times 1/7}$ $\quad \left(\because \left(a^m\right)^n = a^{mn}\right)$

$$\therefore (128)^{1/7} = 2$$

(ii) $7^{1/17}.11^{1/17} = (7\times 11)^{1/17\times 1/17}$ $\quad \left(\because (ab)^m = a^m b^m\right)$

(iii) $\frac{5^2}{5^3} = 5^{2-3}$ $\quad \left(\because \frac{a^m}{a^n} = a^{m-n}\right)$

$$\therefore \frac{5^2}{5^3} = \frac{1}{5} \quad \left(\because a^{-n} = \frac{1}{a^n}\right)$$

(iv) $(0.00032)^{-2/5} = \left(\frac{32}{100000}\right)^{-2/5}$

$$= \left(\frac{100000}{32}\right)^{-2/5} \quad \left(\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right)$$

$$= \left(\frac{10^5}{2^5}\right)^{2/5}$$

$$=\left(\left(\frac{10}{2}\right)^{5}\right)^{2/5} \qquad \left(\because \left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}\right)$$

$$=\left(\frac{10}{2}\right)^{5\times\frac{2}{5}} \qquad \left(\because \left(a^{m}\right)^{n}=a^{m\times n}\right)$$

$$=\left(\frac{10}{2}\right)^{2}=\frac{10^{2}}{2^{2}}=\frac{10\times 10}{2\times 2}=25$$

For competition:

Square root of a binomial quadratic surd:

Step – I:

Express the given surd in the form of $\sqrt{a}\left(b+\sqrt{c}\right)$.

Step – II:

$\sqrt{\sqrt{a}}$ is $\sqrt[4]{a}\,\sqrt{b+\sqrt{c}}$ is $\sqrt{\frac{1}{2}\left[b+\sqrt{b^{2}-c}\right]}+\sqrt{\frac{1}{2}\left[b-\sqrt{b^{2}-c}\right]}$

Step III:

Hence, the required square root is the product of the two square roots obtained.

Important results:

(I) If ‘a’ is a rational number and $\sqrt{b}$ is a surd then,

$$\sqrt{a+\sqrt{b}}=\sqrt{x}+\sqrt{y} \Leftrightarrow \sqrt{a-\sqrt{b}}=\sqrt{x}-\sqrt{y}\,(x>y)$$

(II) If ‘a’ is a rational number and $\sqrt{b}$, $\sqrt{c}$, $\sqrt{d}$ are surds then

$$\sqrt{a+\sqrt{b}+\sqrt{c}+\sqrt{d}}=\sqrt{x}+\sqrt{y}+\sqrt{z}$$

Where $x=\frac{1}{2}\sqrt{\frac{bd}{c}}$, $y=\frac{1}{2}\sqrt{\frac{bc}{d}}$, $z=\frac{1}{2}\sqrt{\frac{cd}{b}}$ and $x+y+z=a$

Cube root of a quadratic surd:

Cube root of quadratic surd can be obtained by using the following principle.

- If ‘a’ and ‘x’ are rational numbers and $\sqrt{b}$, $\sqrt{y}$ are surds then,

$$\sqrt[3]{a+\sqrt{b}}=x+\sqrt{y} \Leftrightarrow \sqrt[3]{a-\sqrt{b}}=x-\sqrt{y}$$