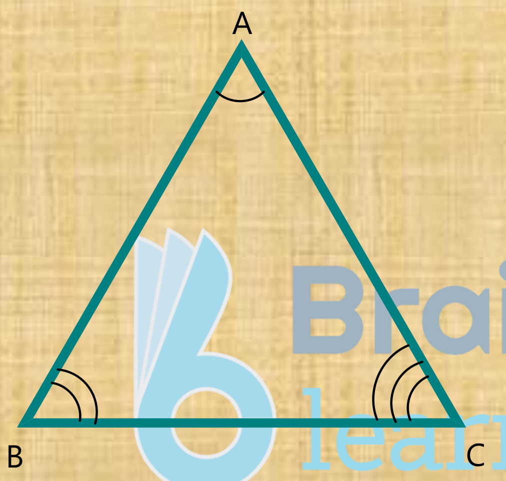
CHAPTER 07

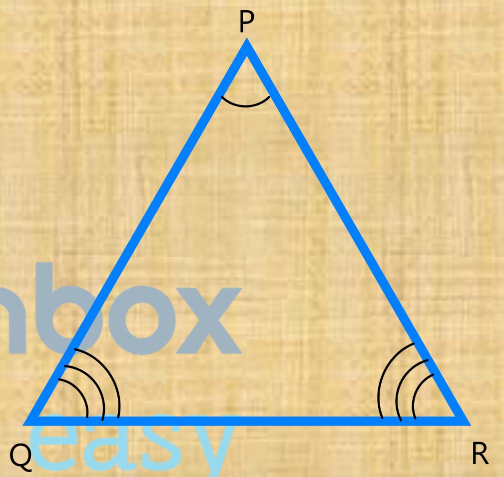
Triangles

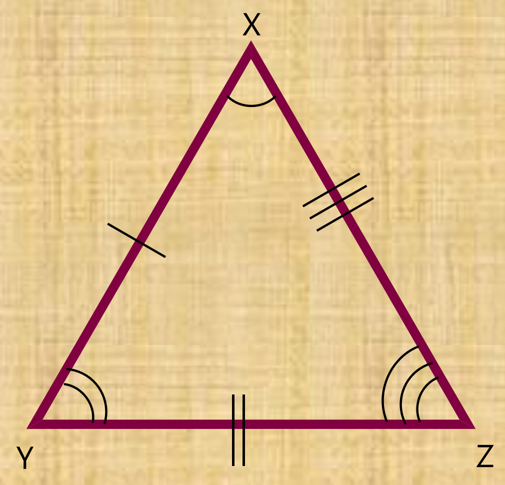
Properties of congruent relation:

- Congruence relation is reflexive.

$\Delta ABC \cong \Delta PQR$







- Congruence relation is symmetric.

In $\Delta ABC \cong \Delta PQR$, then $\Delta PQR \cong \Delta XYZ$

- Congruence relation is transitive.

Suppose there is a third triangle XYZ, which is congruent to ΔPQR.

If $\Delta ABC \cong \Delta PQR$ and $\Delta PQR \cong \Delta XYZ$, then $\Delta ABC \cong \Delta XYZ$.

Criteria for congruence of triangles:

SAS (Side – Side – Side) congruence axiom:

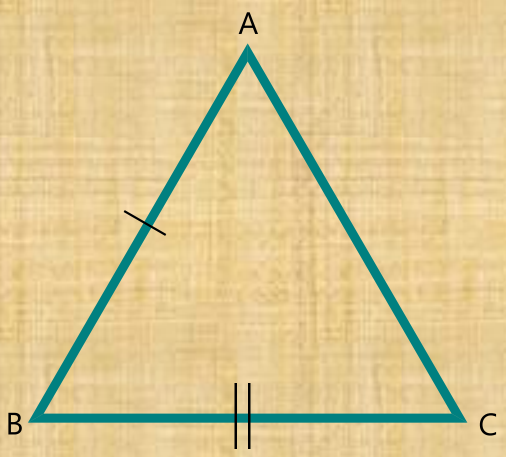
Statement:

Two triangles are congruent if any two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle.

Proof:

If AB = PQ

$\angle B = \angle Q$ and BC = QR then $\Delta ABC \cong \Delta PQR$.

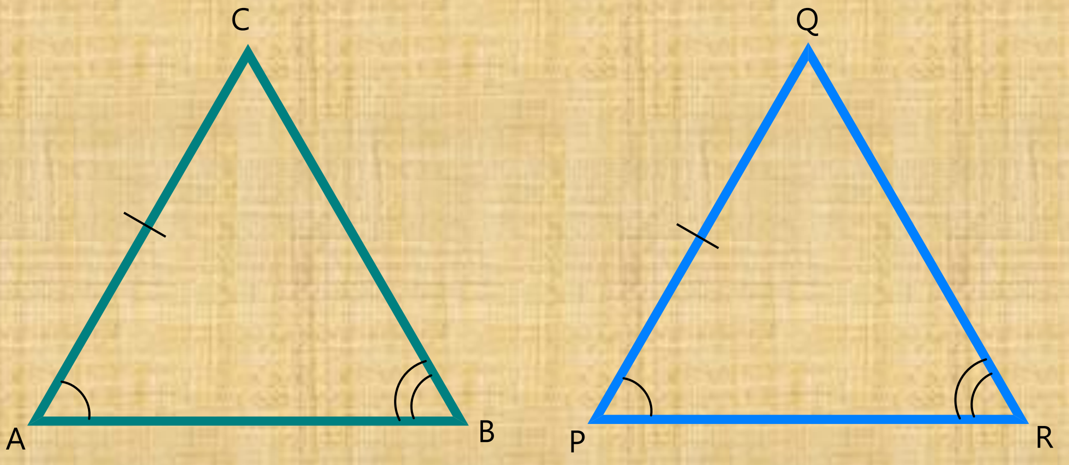


Included angle must be the angle formed by the two equal sides, otherwise the triangles may not be congruent.

ASA (Angle – Side – Angle) congruence axiom:

Statement if any two angles and a non – included side of one triangle are equal to the corresponding two angles and the non – included side of the other triangle then the two triangles are congruent.

Proof:



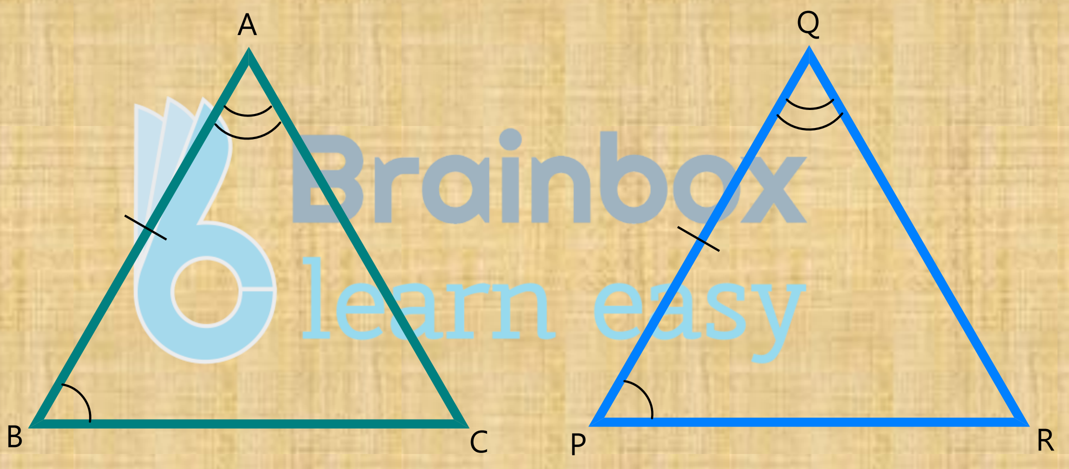
If $\angle A = \angle P$; $\angle B = \angle Q$ and AC = PR then,

$$\Delta ABC \cong \Delta PQR$$

ASA (Angle – Side – Angle) congruence axiom:

Statement:

Two triangles are congruent, if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.



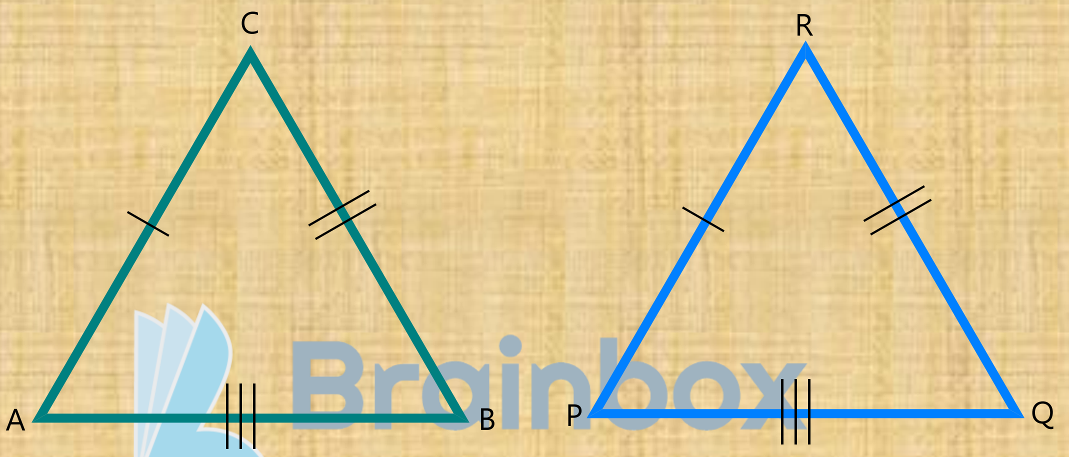
If $\angle B = \angle Q$,

AB = PQ and $\angle A = \angle P$ then $\Delta ABC \cong \Delta PQR$.

SSS (Side – Side – Side) congruence axiom:

Statement:

Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.



Proof:

If AB = PQ

BC = QR and

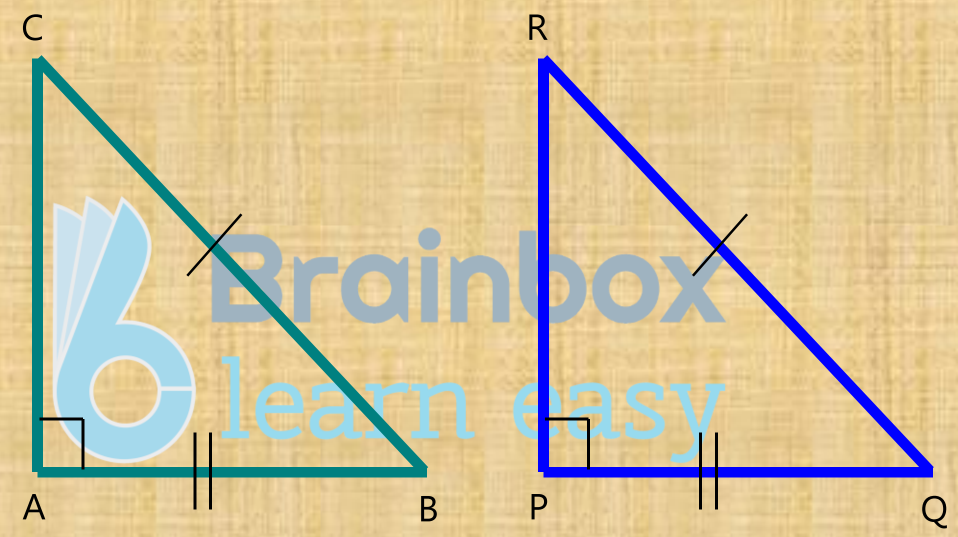
AC = PR then $\Delta ABC \cong \Delta PQR$.

RHS congruence axiom (Right – angle – Hypotenuse – Side):

Statement:

The right-angled triangles are congruent if the hypotenuse and the corresponding side of the other triangle.

Proof:



If $\angle A = \angle P = 90^\circ$,

AB = PQ and BC = QR then $\angle BAC \cong \Delta QPR$.

Here,

AAA is not a criterion for congruence of triangles since the lengths of the sides need not be equal in triangles of equal angles.