CHAPTER 12

Areas Related to Circles

Segment of a circle:

It is defined as the region bounded by a chord and corresponding arc of the circle is known as segment of the circle.

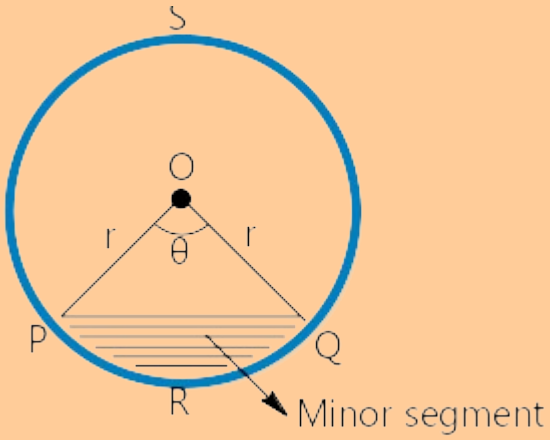
It is divided into two groups.

- Minor segment
- Major segment

Minor segment:

The segment containing the minor arc is called the minor segment.

Here, PRQ is minor segment.



Area of minor segment:

Area of corresponding sector – Area of triangle formed by chord and the radii of the circle

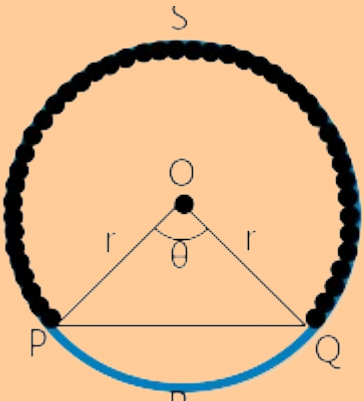
i.e. $= \frac{r^2}{2}\left[\frac{\pi\theta}{180^\circ} - \sin\theta\right]$

Perimeter of minor segment = AB + $\left[\frac{\theta}{360^\circ} \times 2\pi r\right]$

Major segment:

The segment containing the major arc is called major segment.

Here, PSQ is major segment.



Minor segment

Area of major segment = Area of circle – Area of minor segment

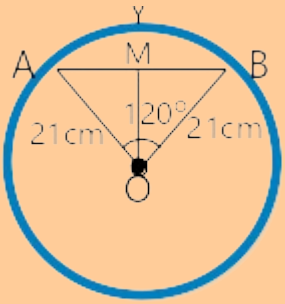
Perimeter of major segment = PQ + $\left[2\pi r - \frac{\theta}{360^\circ} \times \pi r\right]$

- While solving problem, in the given question we have 'segment' means we should find minor segment unless stated otherwise.

Example:

Find the area of the segment AYB as shown in the figure if radius of the circle is 21 cm and $\angle AOB = 120^\circ$ $\left(\text{Use, } \pi = \frac{22}{7}\right)$

Sol.



In the given figure,

Radius (r) = 21 cm

Angle (θ) = 120°

Draw $OM \perp AB$.

$\Rightarrow$OM is perpendicular bisector of AB and bisector of $\angle AOB = 120^\circ$ $(\because OA = OB = 21cm)$

$\Rightarrow$AM = BM = $\frac{1}{2}$AB and $\angle AOM = \angle BOM = 60^\circ$

In right angled triangle OMA, $Cos\theta = \frac{OM}{OA}$

$Cos60^\circ = \frac{OM}{OA}$ and $Sin60^\circ = \frac{AM}{OA}$

$\frac{1}{2} = \frac{OM}{21}$ and $\frac{\sqrt{3}}{2} = \frac{AM}{21}$

$\Rightarrow$OM = $\frac{21}{2}$ = 10.5 cm $\Rightarrow$AM = $\frac{21\sqrt{3}}{2}$ cm

Area of the segment (AYB) =

(Area of the sector OAYB) – (Area of the OAB)

AYB = $\frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} AB \times OM$

$= \left[\frac{120}{360} \times \frac{22}{7} \times 21 \times 21\right] - \left[\frac{1}{2} \times \cancel{2}\left(\frac{21\sqrt{3}}{\cancel{2}}\right) \times 21 \times 21\right]$

$= \left[\frac{1}{\cancel{3}} \times 22 \times \cancel{3} \times \cancel{21}^{7}\right] - \left[\frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}\right]$

$= 462 - \frac{441\sqrt{3}}{4}$

$= \frac{21}{4}\left[88 - 21\sqrt{3}\right] cm^2$

$\therefore$ Area of the segment AYB = $\frac{21}{4}\left[88 - 21\sqrt{3}\right] cm^2$