

Chapter 02

POLYNOMIALS

Division algorithm for polynomials:

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Where remainder is '0' or a polynomial of degree less than that of divisor.

Example:

Divide $3x^2 + x^3 + 3x + 1$ by $x + 1$.

Solution:

Step 1:

Arrange the terms of the dividend and divisor in descending (ascending) order of their degrees.

i.e., $x^3 + 3x^2 + 3x + 1$ and $x + 1$.

$$\begin{array}{r} x+1 \) \ x^3 + 3x^2 + 3x + 1 \ (\ ? \\ \text{(Divisor)} \quad \quad \text{(Dividend)} \quad \quad \text{(Quotient)} \end{array}$$

Step 2:

We get first term of quotient by dividing the first term of dividend with the first term of divisor.

$$\text{i.e., } \frac{x^3}{x} = x^2,$$

So first term of quotient = x^2 .

$$\begin{array}{r} x+1 \) \ x^3 + 3x^2 + 3x + 1 \ (\ x^2 \\ \underline{x^3 + x^2} \end{array}$$

Step 3:

Multiply the divisor by the first term of the quotient and subtract the result from the dividend to get remainder.

i.e.,

$$\begin{array}{r}
 x+1 \overline{) x^3 + 3x^2 + 3x + 1} \\
 \underline{(-) x^3 + (-) x^2} \quad \downarrow \quad \downarrow \\
 2x^2 + 3x + 1 \rightarrow \text{Remainder}
 \end{array}$$

Step 4:

Do the same as in step 2 to get second term of quotient by taking the remainder in step 3 as dividend.

i.e.,

$$\begin{array}{r}
 x+1 \overline{) x^3 + 3x^2 + 3x + 1} \\
 \underline{(-) x^3 + (-) x^2} \\
 2x^2 + 3x + 1 \\
 \underline{(-) 2x^2 + (-) 2x} \quad \downarrow \\
 x + 1 \\
 \underline{(-) x + (-) 1} \\
 0 \rightarrow \text{Remainder}
 \end{array}$$

By step-2: $\frac{2x^2}{x} = \frac{2x \cdot x}{x} = 2x$

By step-3: subtract.

Step 5:

Repeat the above process till we obtain a remainder which is either zero or a polynomial of degree less than that of the divisor. i.e., 0.