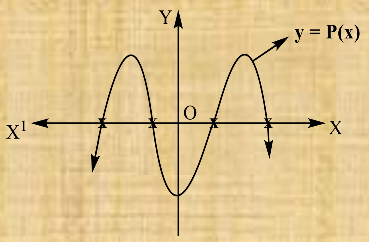
Chapter 02

POLYNOMIALS

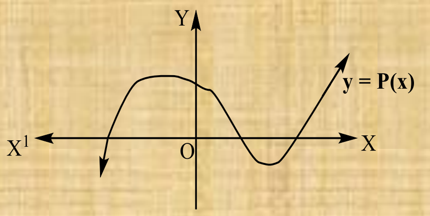
Example-1:

Find the number of zeroes of $P(x)$ for the given graphs.

(i)



(ii)



Solution:

(i) Graph of $y = P(x)$ intersects the $x-$axis at 4 points so, the number of zeroes for the given graph is ‘4’.

(ii) Graph of $y = P(x)$ intersects the $x-$axis at 3 pointsso, the number of zeroes for the given graph is ‘3’.

Example-2:

Find the remainder when $(2x^3 - 3x^2 + 7x - 8)$is divided by $(x-1)$.

Solution:

Let, $\mathrm{P}(x) = 2x^3 - 3x^2 + 7x - 8$

By Remainder theorem,

On dividing $\mathrm{P}(x)$by $(x-1)$, we get

Remainder = $\mathrm{P}(1)$

$\Rightarrow \mathrm{P}(1) = 2(1)^3 - 3(1)^2 + 7(1) - 8 = 2 - 3 + 7 - 8 = -2.$

Hence, the required remainder is $'-2'$.

Example-3:

If $f(x)=\left(2x^3+x^2+bx-6\right)$ leaves a remainder 36 when divided by $(x-3)$, find the value of $'b'$.

Solution:

Given, $f(x)=2x^3+x^2+bx-6 \rightarrow (1)$

$\because f(x)$ divided by $(x-3)$ leaves a remainder 36.

$$\Rightarrow f(3)=36$$

$$\Rightarrow 2(3)^3+(3)^2+b(3)-6=36 \qquad (\because \text{From eq.1})$$

$$\Rightarrow 2(27)+9+3b-6=36$$

$$\Rightarrow 3b+57=36$$

$$\Rightarrow 3b=36+57=-21$$

$$b=\frac{\cancel{-21}^{\,7}}{\cancel{3}}$$

$$\therefore \boxed{b=-7}$$

Example-4:

Show that $(x-1)$ is a factor of $x^3-7x^2+14x-8$. Hence, completely factorise the above expression.

Solution:

Let $\mathrm{P}(x)=x^3-7x^2+14x-8$

By factor theorem,

$(x-1)$ will be a factor of $\mathrm{P}(x)$, if $\mathrm{P}(1)=0$

$$\Rightarrow \mathrm{P}(1)=(1)^3-7(1)^2+14(1)-8$$

$$=\cancel{1}-\cancel{7}+\cancel{14}-\cancel{8}$$

$$\mathrm{P}(1)=0$$

$$\begin{array}{r|l}
 & x^2-6x+8 \\
x-1 & \cancel{x^3}-7x^2+14x-8 \\
 & \underset{(-)}{\cancel{x^3}}\ \underset{(+)}{-x^2} \\
\hline
 & \cancel{-6x^2}+14x-8 \\
 & \underset{(+)}{\cancel{-6x^2}}\ \underset{(-)}{+6x} \\
\hline
 & \cancel{8x-8} \\
 & \cancel{8x-8} \\
\hline
 & 0
\end{array}$$

$\therefore (x-1)$ is a factor of $\mathrm{P}(x)$.

On dividing $\mathrm{P}(x)=x^3-7x^2+14x-8$ by $(x-1)$

We get, Quotient = x^2-6x+8

$$\therefore \mathrm{P}(x)=x^3-7x^2+14x-8$$

$$=(x-1)\left(x^2-6x+8\right)$$

$$=(x-1)\left(x^2-4x-2x+8\right)$$

$$=(x-1)[x(x-4)-2(x-4)]$$

$$=(x-1)(x-4)(x-2)$$

Hence, $x^3-7x^2+14x-8=(x-1)(x-4)(x-2)$.

Example-5:

Find the values of the constants $'a'$ and $'b'$, if $(x-2)$ and $(x+3)$ are both factors of the expression $\left(x^3+ax^2+bx-12\right)$.

Solution:

Let $P(x)=x^3+ax^2+bx-12$

$\because (x-2)$ is a factor of $P(x)$

$\Rightarrow P(2)=0$

$\Rightarrow (2)^3+a(2)^2+b(2)-12=0$

$\Rightarrow (-3)^2+a(-3)^2+b(-3)-12=0$

$\Rightarrow 4a+2b=4$

$\Rightarrow 2a+b=2 \rightarrow (1)$

$\because (x+3)$ is a factor of $P(x)$

$\Rightarrow P(-3)=0$

$\Rightarrow 9a-3b=39$

$\Rightarrow 3a-b=13 \rightarrow (2)$

Add (1) & (2) we get,

$5a=15$

$$\Rightarrow a=\frac{\cancel{15}}{\cancel{5}}=3$$

$\Rightarrow \boxed{a=3}$

Substitute ‘a’ in equation(1),

$$2(3)+b=2$$
$$\Rightarrow 6+b=2$$
$$\Rightarrow b=2-6=-4$$
$$\Rightarrow \boxed{b=-4}$$

Hence, $\boxed{a=3}$ and $\boxed{b=-4}$.

Example-6:

Find the value of $\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)}$.

Solution:

$$\frac{-a(b-c)-b(c-a)-c(a-b)}{(a-b)(b-c)(c-a)}=\frac{0}{(a-b)(b-c)(c-a)}=0.$$

Example-7:

Use the factor theorem to factorise completely (x^3+x^2-4x-4).

Solution:

Let $f(x)=x^3+x^2-4x-4$

The constant term in $f(x)$ is ‘-4’

Its factors are 1, -1, 2, -2, and -4.

By trail and error method,

$f(2)=2^3+2^2-4(2)-4=0$

$\therefore (x-2)$ is a factor of

$f(x)=x^3+x^2-4x-4$

On dividing $f(x)$ by $(x-2)$ we get,

$$\begin{array}{r|l} & x^2+3x+2 \\ \hline x-2 & \cancel{x^3}+x^2-4x-4 \\ & \underline{\cancel{x^3}-2x^2} \\ & \quad (-)\ (+) \\ & \cancel{3x^2}-4x-4 \\ & \underline{\cancel{3x^2}-6x} \\ & \quad (-)\ (+) \\ & \cancel{2x-4} \\ & \underline{\cancel{2x-4}} \\ & 0 \end{array}$$

Quotient = $\left(x^2+3x+2\right)$

$$\therefore x^3+x^2-4x-4=(x-2)\left(x^2+3x+2\right)$$

$$=(x-2)\left(x^2+2x+x+2\right)$$

$$=(x-2)\left(x(x+2)+1(x+2)\right)$$

$$=(x-2)\left((x+2)(x+1)\right)$$

Hence, $x^3+x^2-4x-4=(x+1)(x-2)(x+2)$.