CHAPTER 08

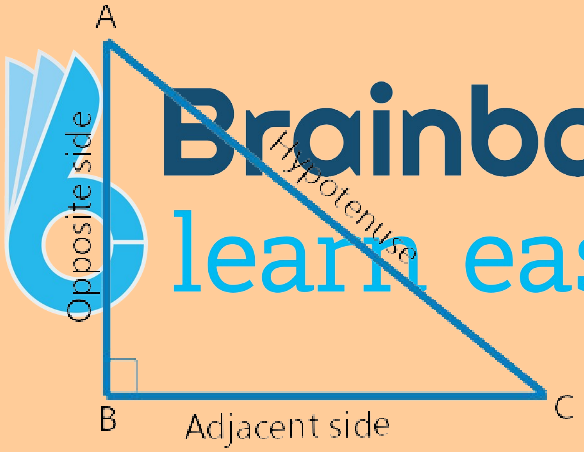
Introduction to Trigonometry

Pythagoras theorem:

In right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In ΔABC,

$$AC^2 = AB^2 + BC^2$$



Note:

The longest side in a right triangle is hypotenuse.

Relation between trigonometric ratios:

- $\text{Cosec}\,\theta = \dfrac{1}{\text{Sin}\,\theta} \Rightarrow \text{Sin}\,\theta = \dfrac{1}{\text{Cosec}\,\theta} \Rightarrow \text{Sin}\,\theta.\text{Cosec}\,\theta = 1$

- $\text{Sec}\theta = \frac{1}{\text{Cos}\theta} \Rightarrow \text{Cos}\theta = \frac{1}{\text{Sec}\theta} \Rightarrow \text{Cos}\theta.\text{Sec}\theta = 1$
- $\text{Cot}\theta = \frac{1}{\text{Tan}\theta} \Rightarrow \text{Tan}\theta = \frac{1}{\text{Cot}\theta} \Rightarrow \text{Tan}\theta.\text{Cot}\theta = 1$
- $\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta}$
- $\text{Tan}\,\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$

Note:

If the product of any two quantities is 1, then they are reciprocal to each other.

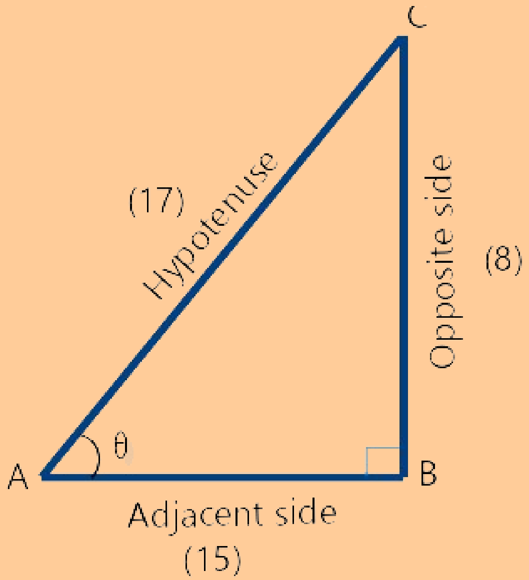
Example:

In ΔABC, right angled at B , if AB = 15 and AC = 17, find all the six trigonometric ratios of angle 'A'.

Sol.

Given that,

$\angle B = 90^\circ$, AB = 15, AC = 17



By Pythagoras theorem, we have

$$\mathbf{AC^2 = AB^2 + BC^2}$$

$$\Rightarrow BC^2 = AC^2 - AB^2$$

$$= (17)^2 - (15)^2$$

$$= 289 - 225$$

$$\Rightarrow BC^2 = 64$$

$$\therefore BC = \sqrt{64} = 8$$

Now, w.r.t to $\sqrt{A}$,

$$\text{Sin} A = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{8}{17}$$

$$\text{Cos}A = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\text{Tan}A = \frac{\text{Opposite side}}{\text{Adjacent Side}} = \frac{8}{15}$$

$$\text{Cosec}A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{17}{8}$$

$$\text{Sec}A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{17}{15}$$

$$\text{Cot}A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{15}{8}$$

Example:

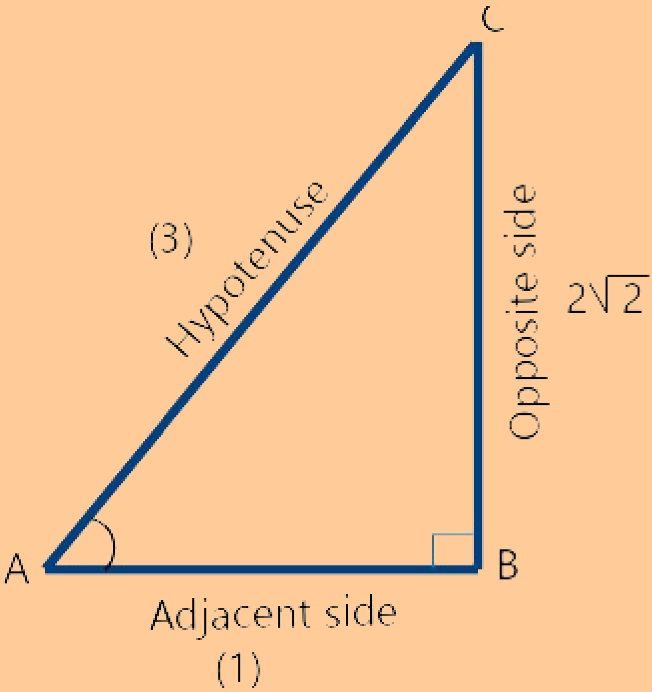
In ΔABC, TanA = $2\sqrt{2}$, find the other T – ratios of angle A.

Sol.

Given that,

$$\text{Tan}A = \frac{2\sqrt{2}}{1} = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

We draw triangle ABC, right angled at 'B' such that AB = 1 and BC = $2\sqrt{2}$.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (1)^2 + (2\sqrt{2})^2$

$= 1 + 8$

$AC^2 = 9$

$AC = \sqrt{9} = 3$

$\therefore$ AC = Hypotenuse = 3

Now,

$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{2\sqrt{2}}{3}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{1}{3}$$

$$\text{Cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{3}{2\sqrt{2}}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = 3$$

Example:

Prove that $\frac{\sec^2\theta - \sin^2\theta}{\tan^2\theta} = \text{Cosec}^2\theta - \cos^2\theta$

Sol.

L.H.S = $\frac{\sec^2\theta - \sin^2\theta}{\tan^2\theta}$

$$= \frac{\frac{1}{\cos^2\theta} - \sin^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{1 - \cos^2\theta.\sin^2\theta}{\cancel{\cos^2}\theta}}{\frac{\sin^2\theta}{\cancel{\cos^2}\theta}}$$

$$= \frac{1 - Cos^2\theta . Sin^2\theta}{Sin^2\theta}$$

$$= \frac{1}{Sin^2\theta} - \frac{Cos^2\theta . Sin^2\theta}{Sin^2\theta}$$

$$= Cosec^2\theta - Cos^2\theta$$

= R.H.S

∴ L.H.S = R.H.S

Hence proved.

Example:

If $Cot\theta = \frac{7}{8}$, evaluate

(i) $Cot^2\theta$

(ii) $\frac{(1+Sin\theta)(1-Sin\theta)}{(1+Cos\theta)(1-Cos\theta)}$

Sol.

Given that,

$$Cot\theta = \frac{7}{8}$$

(i) $\text{Cot}^2\theta = (\text{Cot}\,\theta)^2$

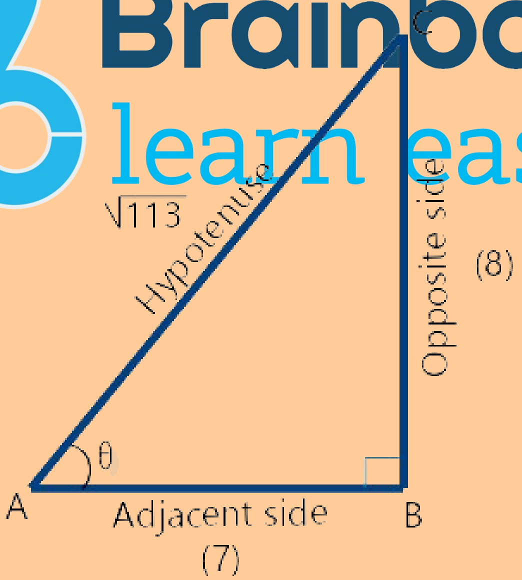
$$= \left(\frac{7}{8}\right)^2$$

$\therefore$ $\text{Cot}^2\theta = \dfrac{49}{64}$

(ii) $\because$ $\text{Cot}\,\theta = \dfrac{\text{Adjacent Side}}{\text{Opposite Side}} = \dfrac{7}{8}$

By Pythagoras theorem, we get

$AC^2 = AB^2 + BC^2$



$\Rightarrow$ $AC^2 = (7)^2 + (8)^2$

$= 49 + 64$

$= 113$

$\Rightarrow AC^2 = 113$

$\Rightarrow AC = \sqrt{113}$

$\therefore \text{Sin}\theta = \frac{O}{H} = \frac{8}{\sqrt{113}}$

$\text{Cos}\theta = \frac{A}{H} = \frac{7}{\sqrt{113}}$

Now,

$$\frac{(1+\text{Sin}\theta)(1-\text{Sin}\theta)}{(1+\text{Cos}\theta)(1-\text{Cos}\theta)} = \frac{(1)^2-(\text{Sin}\theta)^2}{(1)^2-(\text{Cos}\theta)^2}$$

$$(\because (a+b)(a-b) = a^2 - b^2)$$

$$= \frac{1^2-\text{Sin}^2\theta}{1^2-\text{Cos}^2\theta}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{113-64}{\cancel{113}}}{\frac{113-49}{\cancel{113}}}$$

$$= \frac{49}{64}$$

Hence,

$$\frac{(1+\text{Sin}\,\theta)(1-\text{Sin}\,\theta)}{(1+\text{Cos}\,\theta)(1-\text{Cos}\,\theta)} = \frac{49}{64}$$