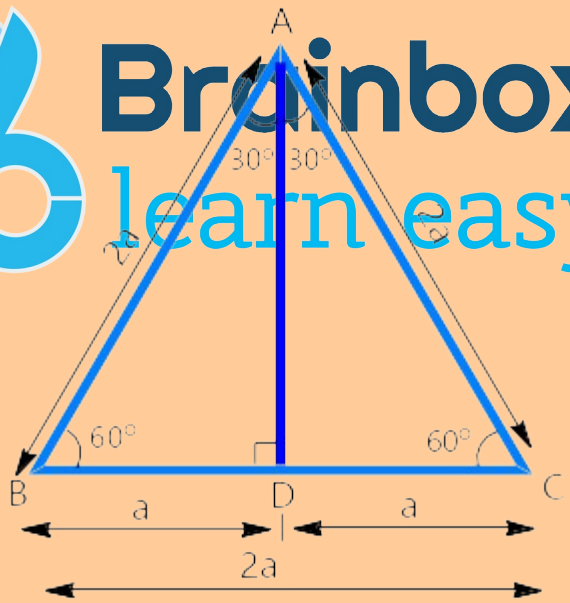
CHAPTER 08

Introduction to Trigonometry

Trigonometric ratios of 30°:

To get the trigonometric ratios of angles 30° and 60°. Let us consider an equilateral triangle ABC of 2a units each and split into two right – angled triangles by draw AD perpendicular to BC.

∵ ABC is an equilateral triangle.



∴ AD is the bisector of $\sqrt{A}$ and D is the midpoint of BC

[∵ $\Delta ADC \cong \Delta ADB$ by R.H.S congruence rule]

$\Rightarrow \angle BAD = \angle CAD = \frac{\angle A}{2} = \frac{60^\circ}{2} = 30^\circ$ and BD = CD = $\frac{1}{2}$ BC = a units

Now, ΔABD is a right angled at 'D' i.e. $\angle D = 90^\circ$ in which AB = 2a and BD = a.

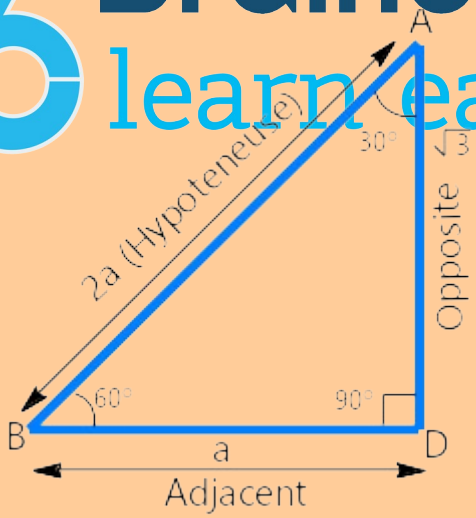
By Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + (a)^2$

$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$

$\therefore AD = \sqrt{3}a$



In ΔABD, AB = 2a, BD = a, AD = $\sqrt{3}a$ and $\angle A$ = 30°.

$\sin 30^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{2a} = \frac{1}{2}$

$$\Rightarrow \text{Cosec}30^\circ = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{2\cancel{a}}{\cancel{a}} = 2$$

$$\text{Cos}30^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\sqrt{3}\cancel{a}}{2\cancel{a}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \text{Sec}30^\circ = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{2\cancel{a}}{\sqrt{3}\cancel{a}} = \frac{2}{\sqrt{3}}$$

$$\text{Tan}30^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\cancel{a}}{\sqrt{3}\cancel{a}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \text{Cot}30^\circ = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{\sqrt{3}\cancel{a}}{\cancel{a}} = \sqrt{3}$$

Summary:

$$\text{Sin}30^\circ = \frac{1}{2};\ \text{Cos}30^\circ = \frac{\sqrt{3}}{2};\ \text{Tan}30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Cosec}30^\circ = 2;\ \text{Sec}30^\circ = \frac{2}{\sqrt{3}};\ \text{Cot}30^\circ = \sqrt{3}$$

Trigonometric ratios of 60°:

In ΔABD, $\angle B = 60^\circ$

$$\text{Sin}60^\circ = \frac{O}{H} = \frac{\sqrt{3}\cancel{a}}{2\cancel{a}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \text{Cosec}\,60^\circ = \frac{H}{O} = \frac{2\cancel{a}}{\sqrt{3}\cancel{a}} = \frac{2}{\sqrt{3}}$$

$$\text{Cos}\,60^\circ = \frac{A}{H} = \frac{\cancel{a}}{2\cancel{a}} = \frac{1}{2}$$

$$\Rightarrow \text{Sec}\,60^\circ = \frac{H}{A} = \frac{2\cancel{a}}{\cancel{a}} = 2$$

$$\text{Tan}\,60^\circ = \frac{O}{A} = \frac{\sqrt{3}\cancel{a}}{\cancel{a}} = \sqrt{3}$$

$$\Rightarrow \text{Cot}\,60^\circ = \frac{A}{O} = \frac{\cancel{a}}{\sqrt{3}\cancel{a}} = \frac{1}{\sqrt{3}}$$

Summary:

- $\text{Sin}\,60^\circ = \frac{\sqrt{3}}{2}$; $\text{Cos}\,60^\circ = \frac{1}{2}$; $\text{Tan}\,60^\circ = \sqrt{3}$
- $\text{Cosec}\,60^\circ = \frac{2}{\sqrt{3}}$; $\text{Sec}\,60^\circ = 2$; $\text{Cot}\,60^\circ = \frac{1}{\sqrt{3}}$