CHAPTER 08

Introduction to Trigonometry

Trigonometric ratios of complementary angles:

Complementary angles:

Two angles are said to be complementary when their sum is equal to 90°. Thus 'θ' is any acute angle, then its complementary angle will be $90^\circ - \theta$.

Examples:

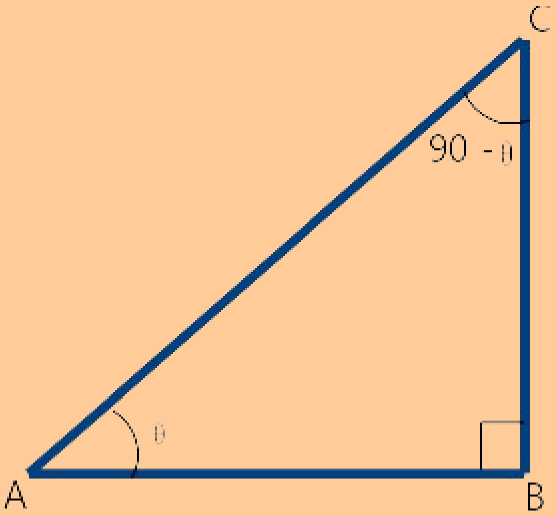
(45° and 45°), (30° and 60°)

T – Ratios of complementary angles – proof:

Trigonometric ratios for $(90^\circ - \theta)$:

Consider a right – angled triangle ABC, right vertex at 'B'.

Let $\angle A = \theta$ then $\angle C = 90^\circ - \theta$



In ΔABC, with reference to $\angle A = \theta$.

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC} \quad \text{........ (1)}$$

$$\Rightarrow \operatorname{cosec}(\theta) = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{AC}{BC} \quad \text{........ (4)}$$

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC} \quad \text{........ (2)}$$

$$\Rightarrow \sec(\theta) = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{AC}{AB} \quad \text{........ (5)}$$

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB} \quad \text{........ (3)}$$

$$\Rightarrow \cot(\theta) = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{AB}{BC} \quad \text{........... (6)}$$

In ΔABC, with reference to $\angle C = 90^\circ - \theta$

$$\mathrm{Sin}\left(90^\circ - \theta\right) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC} = \mathrm{Cos}\left(\theta\right)$$

$$\left(\because \text{ from } \left(2\right)\right)$$

$$\mathrm{Cos}\left(90^\circ - \theta\right) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC} = \mathrm{Sin}\left(\theta\right)$$

$$\left(\because \text{ from } \left(1\right)\right)$$

$$\mathrm{Tan}\left(90^\circ - \theta\right) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC} = \mathrm{Cot}\left(\theta\right)$$

$$\left(\because \text{ from } \left(6\right)\right)$$

$$\mathrm{Cosec}\left(90^\circ - \theta\right) = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{AC}{AB} = \mathrm{Sec}\left(\theta\right)$$

$$\left(\because \text{ from } \left(5\right)\right)$$

$$\mathrm{Sec}\left(90^\circ - \theta\right) = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{AC}{BC} = \mathrm{Cosec}\left(\theta\right)$$

$$\left(\because \text{ from } \left(4\right)\right)$$

$$\mathrm{Cot}\left(90^\circ - \theta\right) = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{BC}{AB} = \mathrm{Tan}\left(\theta\right)$$

$\left(\because \text{from}\left(3\right)\right)$

Table of complementary angles:

$\text{Sin}\left(90^\circ-\theta\right)=\text{Cos}\theta$
$\text{Cos}\left(90^\circ-\theta\right)=\text{Sin}\theta$
$\text{Tan}\left(90^\circ-\theta\right)=\text{Cot}\theta$
$\text{Cosec}\left(90^\circ-\theta\right)=\text{Sec}\theta$
$\text{Sec}\left(90^\circ-\theta\right)=\text{Cosec}\theta$
$\text{Cot}\left(90^\circ-\theta\right)=\text{Tan}\theta$