CHAPTER 11

Trigonometry

Trigonometric identities:

An identity is that mathematical equation, which is true for all the value of the variables in the equation.

Example:

$(a + b)^2 = a^2 + b^2 + 2ab$ is an identity.

In the same way, an identity equation having trigonometric ratios of an angle is called trigonometric identity. It is true for all the values of the angles 'θ' involved in it.

Example:

$\text{Tan}\theta = \frac{1}{\text{Cot}\theta}$, is a trigonometric identity, as it holds for all values of 'θ' except $\theta = 0°$ and 90°.

Thus, $\text{Tan}\theta = \frac{1}{\text{Cot}\theta}$, for all $0° < \theta < 90°$.

Simple trigonometric identities and their proof:

Basically, there are three identities.

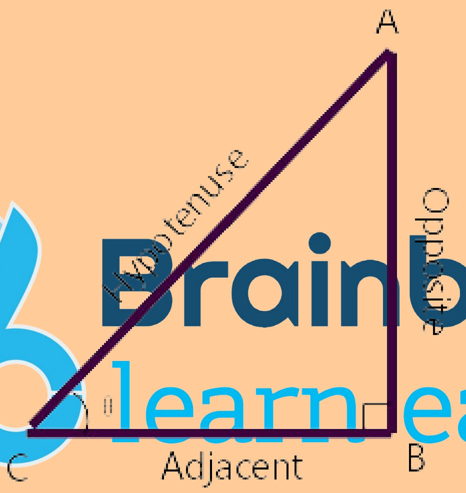
I) $\sin^2\theta + \cos^2\theta = 1$

II) $\sec^2\theta - \tan^2\theta = 1$

III) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

I) $\sin^2\theta + \cos^2\theta = 1$

Proof:



Consider, in ΔABC right angle at B, we have

$AB^2 + BC^2 = AC^2$

Dividing by AC^2 on both sides

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{\cancel{AC^2}}{\cancel{AC^2}}$$

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

$$\left(\frac{\text{Opposite}}{\text{Hypotenuse}}\right)^2 + \left(\frac{\text{Adjacent}}{\text{Hypotenuse}}\right)^2 = 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

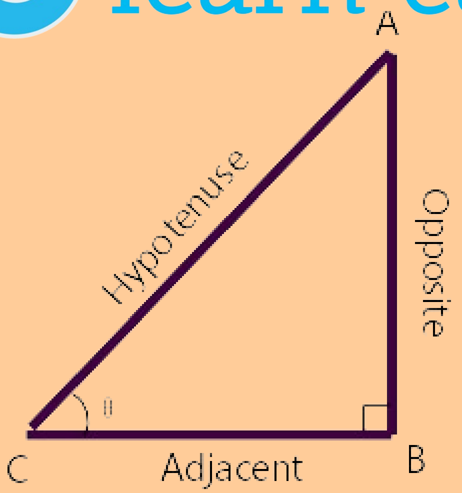
II) $\sec^2\theta - \tan^2\theta = 1$

Proof:

Consider, in ΔABC right angle at B, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 - AB^2 = BC^2$



Dividing by BC^2 on both side

$$\Rightarrow \frac{AC^2}{BC^2} - \frac{AB^2}{BC^2} = \frac{\cancel{BC^2}}{\cancel{BC^2}}$$

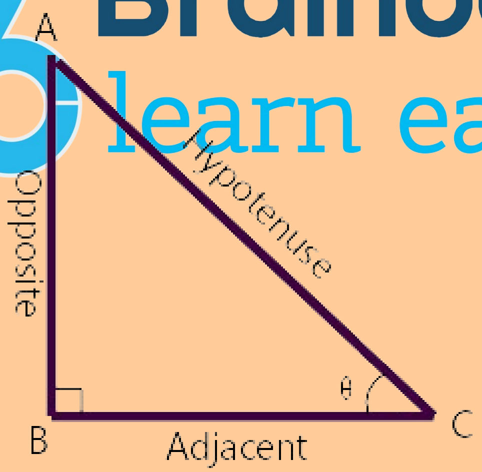
$$\left(\frac{AC}{BC}\right)^2 - \left(\frac{AB}{BC}\right)^2 = 1$$

$$\left(\frac{\text{Hypotenuse}}{\text{Adjacent}}\right)^2 - \left(\frac{\text{Opposite}}{\text{Adjacent}}\right)^2 = 1$$

$$\boxed{\sec^2\theta - \tan^2\theta = 1}$$

III) $\operatorname{Cosec}^2\theta - \cot^2\theta = 1$

Proof:



Consider a right angled at 'B' in ΔABC, we have

$AC^2 = AB^2 + BC^2$

$AC^2 - BC^2 = AB^2$

Dividing by AB^2 on the both sides,

$$\frac{AC^2}{AB^2} - \frac{BC^2}{AB^2} = \frac{\cancel{AB^2}}{\cancel{AB^2}}$$

$$\left(\frac{AC}{AB}\right)^2 - \left(\frac{BC}{AB}\right)^2 = 1$$

$$\left(\frac{\text{Hypotenuse}}{\text{Opposite}}\right)^2 - \left(\frac{\text{Adjacent}}{\text{Opposite}}\right)^2 = 1$$

$$\boxed{\text{Cosec}^2\theta - \text{Cot}^2\theta = 1}$$

Relations:

- $\text{Sin}^2\theta + \text{Cos}^2\theta = 1$ (or) $\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$ (or)

  $\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$

- $\text{Sec}^2\theta - \text{Tan}^2\theta = 1$ (or) $\text{Sec}^2\theta = 1 + \text{Tan}^2\theta$ (or)

  $\text{Tan}^2\theta = \text{Sec}^2\theta - 1$

  $\Rightarrow (\text{Sec}\theta + \text{Tan}\theta)(\text{Sec}\theta - \text{Tan}\theta) = 1$ (or)

  $\text{Sec}\theta + \text{Tan}\theta = \dfrac{1}{\text{Sec}\theta - \text{Tan}\theta}$

- $\text{Cosec}^2\theta - \text{Cot}^2\theta = 1 \Rightarrow (\text{Cosec}\theta - \text{Cot}\theta)(\text{Cosec}\theta + \text{Cot}\theta) = 1$

  (or) (or)

$$\text{Cosec}^2\theta = 1 + \text{Cot}^2\theta$$

(or)

$$\text{Cot}^2\theta = \text{Cosec}^2\theta - 1$$

$$\text{Cosec}\,\theta + \text{Cot}\,\theta = \frac{1}{\text{Cosec}\,\theta - \text{Cot}\,\theta}$$