

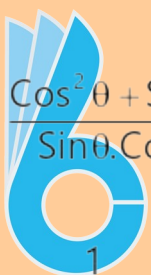
CHAPTER 11**Trigonometry****Example:**

Show that $\text{Cot}\theta + \text{Tan}\theta = \text{Sec}\theta.\text{Cosec}\theta$.

Sol.

$$\text{L.H.S} = \text{Cot}\theta + \text{Tan}\theta$$

$$= \frac{\text{Cos}\theta}{\text{Sin}\theta} + \frac{\text{Sin}\theta}{\text{Cos}\theta}$$


$$= \frac{\text{Cos}^2\theta + \text{Sin}^2\theta}{\text{Sin}\theta.\text{Cos}\theta}$$

$$= \frac{1}{\text{Sin}\theta.\text{Cos}\theta} \quad (\because \text{Sin}^2\theta + \text{Cos}^2\theta = 1)$$

$$= \left(\frac{1}{\text{Sin}\theta} \right) \cdot \left(\frac{1}{\text{Cos}\theta} \right)$$

$$= \text{Cosec}\theta.\text{Sec}\theta$$

$$= \text{R.H.S}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

Hence proved.

Example:

Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{Cosec}\theta + \cot\theta$; $0 \leq \theta \leq 90^\circ$.

Sol.

$$\text{L.H.S} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

Multiply numerator and denominator by $\sqrt{1+\cos\theta}$

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2 - (\cos\theta)^2}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{Cosec}\theta + \cot\theta = \text{R.H.S}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

Hence proved.

Example:

Prove that $\frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \frac{1 - \sin\theta}{\cos\theta}$.

Sol.

$$\text{L.H.S} = \frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta}$$

Multiplying numerator and denominator by $(\sec\theta - \tan\theta)$, we get

$$\begin{aligned} &= \frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} \\ &= \frac{(1 + \sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta) + (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)} \\ &= \frac{(1 + \sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta + (\sec^2\theta - \tan^2\theta)} \\ &= \frac{(1 + \sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{\cancel{\sec\theta - \tan\theta} + 1} \end{aligned}$$

$$(\because \sec^2 \theta - \tan^2 \theta = 1)$$

$$= \sec \theta - \tan \theta$$

$$= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1 - \sin \theta}{\cos \theta}$$

$$= \text{R.H.S}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

Hence, proved.

