CHAPTER 11

Trigonometry

Example:

Show that $\text{Cot}\theta + \text{Tan}\theta = \text{Sec}\,\theta.\text{Cosec}\theta$.

Sol.

L.H.S = $\text{Cot}\theta + \text{Tan}\theta$

$$= \frac{\text{Cos}\theta}{\text{Sin}\theta} + \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$$= \frac{\text{Cos}^2\theta + \text{Sin}^2\theta}{\text{Sin}\theta.\text{Cos}\theta}$$

$$= \frac{1}{\text{Sin}\theta.\text{Cos}\theta} \qquad \left(\because \text{Sin}^2\theta + \text{Cos}^2\theta = 1\right)$$

$$= \left(\frac{1}{\text{Sin}\theta}\right).\left(\frac{1}{\text{Cos}\theta}\right)$$

$$= \text{Cosec}\theta.\text{Sec}\theta$$

= R.H.S

$\therefore$ L.H.S = R.H.S

Hence proved.

Example:

Prove that $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \operatorname{Cosec}\theta + \operatorname{Cot}\theta$; $0 \le \theta \le 90^\circ$.

Sol.

L.H.S = $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$

Multiply numerator and denominator by $\sqrt{1+\cos\theta}$

$= \sqrt{\dfrac{1+\cos\theta}{1-\cos\theta} \cdot \dfrac{1+\cos\theta}{1+\cos\theta}}$

$= \sqrt{\dfrac{(1+\cos\theta)^2}{(1)^2 - (\cos\theta)^2}}$

$= \sqrt{\dfrac{(1+\cos\theta)^2}{\sin^2\theta}}$ $\quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{1+\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \operatorname{Cosec}\theta + \operatorname{Cot}\theta$ = R.H.S

$\therefore$ L.H.S = R.H.S

Hence proved.

Example:

Prove that $\frac{1+\text{Sec}\theta-\text{Tan}\theta}{1+\text{Sec}\theta+\text{Tan}\theta}=\frac{1-\text{Sin}\theta}{\text{Cos}\theta}$.

Sol.

L.H.S = $\frac{1+\text{Sec}\theta-\text{Tan}\theta}{1+\text{Sec}\theta+\text{Tan}\theta}$

Multiplying numerator and denominator by $(\text{Sec}\,\theta-\text{Tan}\,\theta)$, we get

$$=\frac{1+\text{Sec}\theta-\text{Tan}\theta}{1+\text{Sec}\theta+\text{Tan}\theta}\times\frac{\text{Sec}\theta-\text{Tan}\theta}{\text{Sec}\theta-\text{Tan}\theta}$$

$$=\frac{(1+\text{Sec}\theta-\text{Tan}\theta)(\text{Sec}\theta-\text{Tan}\theta)}{(\text{Sec}\theta-\text{Tan}\theta)+(\text{Sec}\theta+\text{Tan}\theta)(\text{Sec}\theta-\text{Tan}\theta)}$$

$$=\frac{(1+\text{Sec}\theta-\text{Tan}\theta)(\text{Sec}\theta-\text{Tan}\theta)}{\text{Sec}\,\theta-\text{Tan}\,\theta+(\text{Sec}^2\theta-\text{Tan}^2\theta)}$$

$$=\frac{\cancel{(1+\text{Sec}\theta-\text{Tan}\theta)}(\text{Sec}\theta-\text{Tan}\theta)}{\cancel{\text{Sec}\theta-\text{Tan}\theta+1}}$$

$\left(\because \sec^2\theta - \tan^2\theta = 1\right)$

$= \sec\theta - \tan\theta$

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1-\sin\theta}{\cos\theta}$

= R.H.S

$\therefore$ L.H.S = R.H.S

Hence, proved.