CHAPTER 08

Introduction to Trigonometry

Example:

Evaluate

(i) $\dfrac{\text{Sec}35^\circ}{\text{Cosec}55^\circ}$

(ii) $\dfrac{\text{Sin}58^\circ}{\text{Cos}32^\circ}$

Sol.

(i) $\text{Cosec}\,A = \text{Sec}\left(90^\circ - A\right)$

$\text{Cosec}55^\circ = \text{Sec}\left(90^\circ - 55^\circ\right)$

$\text{Cosec}55^\circ = \text{Sec}35^\circ$ .... (1)

Now,

$$\frac{\text{Sec}35^\circ}{\text{Cosec}55^\circ} = \frac{\cancel{\text{Sec}35^\circ}}{\cancel{\text{Sec}35^\circ}}$$ ($\because$ From (1))

$$\therefore \frac{\text{Sec}35^\circ}{\text{Cosec}55^\circ} = 1$$

(ii) $\text{Sin}A = \text{Cos}(90^\circ - A)$

$\text{Sin}58^\circ = \text{Cos}(90^\circ - 58^\circ)$

$\text{Sin}58^\circ = \text{Cos}32^\circ$ .... (1)

Now,

$$\frac{\text{Sin}58^\circ}{\text{Cos}32^\circ} = \frac{\cancel{\text{Cos}32^\circ}}{\cancel{\text{Cos}32^\circ}} \qquad (\because \text{From}(1))$$

$$\therefore \frac{\text{Sin}58^\circ}{\text{Cos}32^\circ} = 1$$

Example:

Express $\text{Sin}81^\circ + \text{Tan}81^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Sol.

$\text{Sin}81^\circ = \text{Sin}(90^\circ - 9^\circ) = \text{Cos}9^\circ$

$(\because \text{Sin}(90^\circ - \theta) = \text{Cos}\theta)$

$\text{Tan}81^\circ = \text{Tan}(90^\circ - 9^\circ) = \text{Cot}9^\circ$

$(\because \text{Tan}(90^\circ - \theta) = \text{Cot}\theta)$

Then,

$$\sin 81^\circ + \tan 81^\circ = \cos 9^\circ + \cot 9^\circ$$

Example:

Find the value of (Tan1°Tan2°Tan3°......Tan89°).

Sol.

Given expression,

Tan1° .Tan2°.Tan3°.Tan4°......Tan89°

= Tan1° .Tan2°.Tan3°.Tan4° .....Tan45°.......Tan87°.Tan88°.Tan89°

= Tan(90° – 89°).Tan(90° – 88°).Tan(90° – 87°)

.......Tan45°....Tan87°Tan88°Tan89°.

= Cot89°.Cot88°.Cot87°......Tan45°....Tan88°.Tan89°.

$$\left(\because \tan\left(90^\circ - \theta\right) = \cot\theta\right)$$

= (Cot89°.Tan89°) (Cot88°.Tan88°)(Cot87°.Tan87°)......

(Cot46°Tan46°).Tan45°

$$= \left(\frac{1}{\tan 89^\circ}.\tan 89^\circ\right)\left(\frac{1}{\tan 88^\circ}.\tan 88^\circ\right)\left(\frac{1}{\tan 87^\circ}.\tan 87^\circ\right).......$$

$$.........\left(\frac{1}{\tan 46^\circ}.\tan 46^\circ\right).\tan 45^\circ$$

$= 1.1.1......1.1$ $\left(\because \text{Cot}\theta = \frac{1}{\text{Tan}\theta}\right)$ & $\left(\text{Tan}45^\circ = 1\right)$

Hence, **Tan1°Tan2°Tan3°......Tan89° = 1**

Example:

If $\text{Sin}3A = \text{Cos}\left(A - 26^\circ\right)$, where 3A is an acute angle, find the value of 'A'.

Sol.

We have,

$\text{Sin}3A = \text{Cos}\left(A - 26^\circ\right)$

$\Rightarrow \text{Cos}\left(90^\circ - 3A\right) = \text{Cos}\left(A - 26^\circ\right)$

$\Rightarrow 90^\circ - 3A = A - 26^\circ$

($\because$ Both $90^\circ - 3A$ and $A - 26^\circ$ are acute angles)

$\Rightarrow 90^\circ + 26^\circ = A + 3A$

$\Rightarrow 4A = 116^\circ$

$\Rightarrow A = \frac{\cancel{116}}{\cancel{4}}$

$\Rightarrow A = 29^\circ$

Example:

If A, B and C are interior angles of a triangle ABC, then show

that $\text{Tan}\left(\frac{A+B}{2}\right) = \text{Cot}\left(\frac{C}{2}\right)$.

Sol.

Given A, B, C are angles of triangle ABC then

$A + B + C = 180°$

$\Rightarrow A + B = 180° - C$

On dividing the above equation by '2' on both sides, we get

$$\Rightarrow \frac{A+B}{2} = \frac{180° - C}{2}$$

$$\Rightarrow \frac{A+B}{2} = \frac{\cancel{180°}}{\cancel{2}} - \frac{C}{2}$$

$$\Rightarrow \frac{A+B}{2} = 90° - \frac{C}{2}$$

On taking 'Tan' ratio on both sides

$$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right)=\tan\left(90^\circ-\frac{C}{2}\right)$$

$$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right)=\cot\left(\frac{C}{2}\right) \qquad \left(\because \tan(90-\theta)=\cot\theta\right)$$

Hence, proved.