CHAPTER 06

Triangles

Theorem:

Basic proportionality theorem (OR) Thales theorem (B.P.T):

Statement:

If a line is drawn parallel to one side of a triangle intersect the other two sides at distinct points, and then the other two sides are divided in the same ratio.

Given:

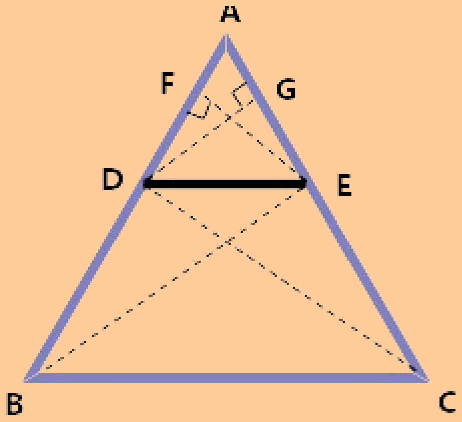
A triangle ABC in which DE || BC intersecting AB and AC at D and E respectively.

To Prove:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction:

Join BE, CD and draw $EF \perp BA$ and $DG \perp CA$.



Since, EF is perpendicular to AB.

Therefore, EF is the height for both triangles ADE and DBE.

Area $(\Delta ADE) = \frac{1}{2} \times (\text{base}) \times (\text{height})$

Area $(\Delta ADE) = \frac{1}{2} \times (AD \times EF)$

(And)

Area $(\Delta DBE) = \frac{1}{2} \times (\text{base}) \times (\text{height})$

Area $(\Delta DBE) = \frac{1}{2} \times (DB \times EF)$

$\therefore \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DBE)} = \frac{\frac{1}{2}(AD \times EF)}{\frac{1}{2}(DB \times EF)} = \frac{AD}{DB}$ ….. (1)

Similarly,

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DEC)} = \frac{\frac{1}{2}(AE \times DG)}{\frac{1}{2}(EC \times DG)} = \frac{AE}{EC}$$ ...... (2)

($\because$ $DG \perp AC$, $\therefore$ DG is height of both triangles ADE and DEC)

Since, ΔDBE and ΔDEC are on the same base DE and between the same parallel lines DE and BC.

$\therefore \text{Area}(\Delta DBE) = \text{Area}(\Delta DEC)$

$$\Rightarrow \frac{1}{\text{Area}(\Delta DBE)} = \frac{1}{\text{Area}(\Delta DEC)}$$

(Taking reciprocal on both sides)

$$\Rightarrow \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DBE)} = \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DEC)}$$

(Multiplying both sides by area ΔADE)

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$ (From (i) and (ii))

Hence, proved.

Corollary:

If in a ΔABC, a line DE || BC, intersects AB in D and AC in E, then

- $\frac{AB}{AD} = \frac{AC}{AE}$
- $\frac{AB}{DB} = \frac{AC}{EC}$

Theorem:

Converse of the BPT (Thales theorem):

Statement:

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Given:

ΔABC and a line 'l' intersecting AB and AC at 'D' and 'E' respectively. Such that $\frac{AD}{DB} = \frac{AE}{EC}$

To Prove:

DE || BC (OR) l || BC.

Proof:

If possible, let DE is not parallel to BC. Then, there must be another line parallel to BC. Let DF || BC. Since DF || BC.

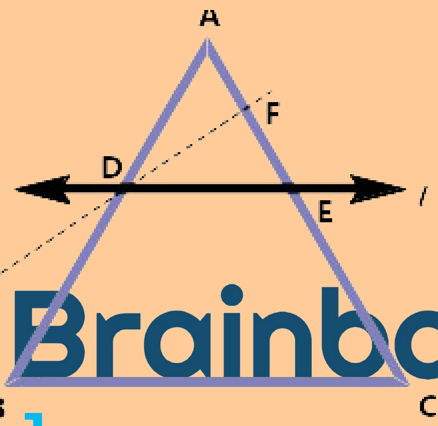
$\therefore$ By basic proportionality theorem, we have

$\frac{AD}{DB} = \frac{AF}{FC}$ ...... (1)

But,

$\frac{AD}{DB} = \frac{AE}{EC}$ (Given) ...... (2)

From (1) and (2),



$\frac{AF}{FC} = \frac{AE}{EC}$

$\Rightarrow \frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$ (Adding 1 on both sides)

$\Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$

$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC}$

$\Rightarrow$ FC = EC

This is possible only when 'F' and 'E' coincide.

i.e. DF is the line 'l' itself.

Hence, $l \parallel BC$.

Note:

If in a ΔABC, D and E are points on AB and AC respectively.

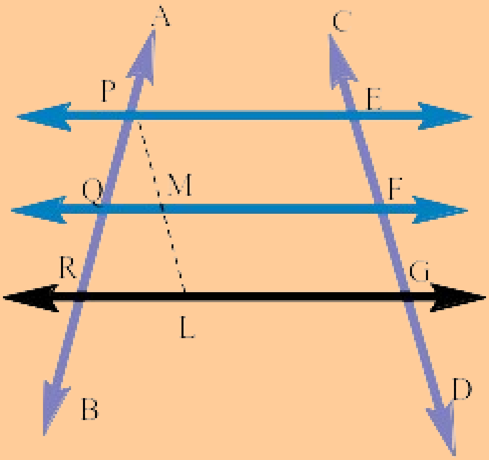
Such that,

(i) $\frac{AB}{AD} = \frac{AC}{AE}$

(ii) $\frac{AB}{BD} = \frac{AC}{EC}$, then DE || BC.

Some important theorems based on BPT:

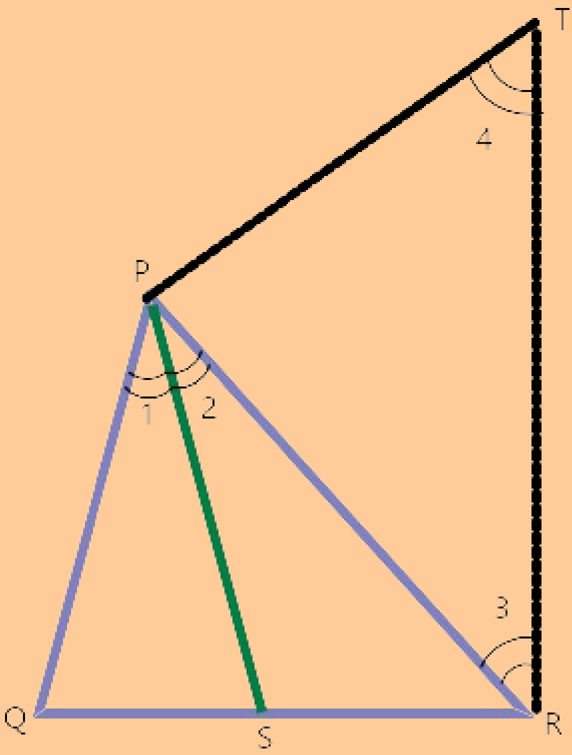
- If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.



$$\frac{PQ}{QR} = \frac{EF}{FG}$$

Angle bisector theorem:

The internal bisector of an angle of a triangle divides the opposite side in two segment that are proportional to the other two sides of the triangle.



$$\frac{QS}{SR} = \frac{PQ}{PR}$$

Given:

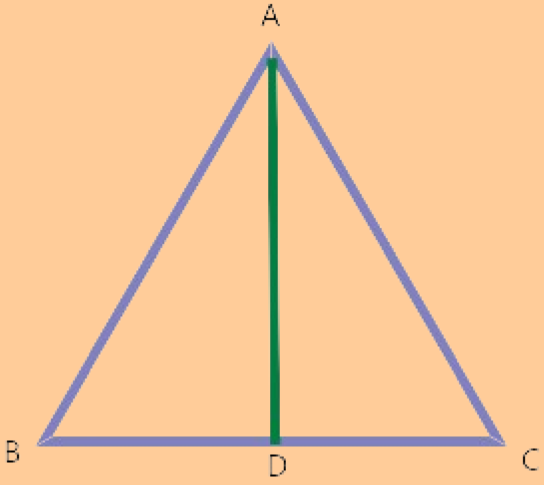
ΔABC in which AD, the bisector of $\angle A$ meets BC in D.

To prove:

$$\frac{QS}{SR} = \frac{PQ}{PR}$$

Constructions:

Draw RT || SP meeting QP produced at T.



Proof:

$\because$ SP || RT

$\angle 2 = \angle 3$ (Alternate into interior angle)

(And)

$\angle 1 = \angle 4$ (Corresponding $\angle$S)

$\Rightarrow \angle 3 = \angle 4$

So, PT = PR

Now, In ΔQRT, SP || RT

$\therefore \frac{QS}{SR} = \frac{PQ}{PT}$

$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$ ($\because$ PT = PR)

Hence, $\frac{QS}{SR} = \frac{PQ}{PR}$