Chapter 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Introduction:

In this chapter, we will learn about a new kind of number called as Complex number. Let us consider an equation $x^2 = 1$, then we happily conclude that 1, -1 are solutions of above equation but think of $x^2 = -1$.

Now what value of x will satisfy this? Here comes the introduction of complex numbers. For any real number, square of any real number can never be negative. So, we introduce a complex number called as iota (i) for which $i^2 = -1$. Here 'i' is a complex number.

Complex Number:

Any number which is expressed in the form z = x + iy where $x, y \in R$ and $i^2 = -1$ or $i = \sqrt{-1}$ or $z = \{x + iy, x, y \in R, i = \sqrt{-1}\}$.

For example, 2 + i.0, 0 + i.2, 2 + i, 1 + 2i are complex numbers.

For any complex number z = x + iy, x is called as real part of z or Re(z) and y is called as imaginary part of z (or) $\text{Im}(z)$.

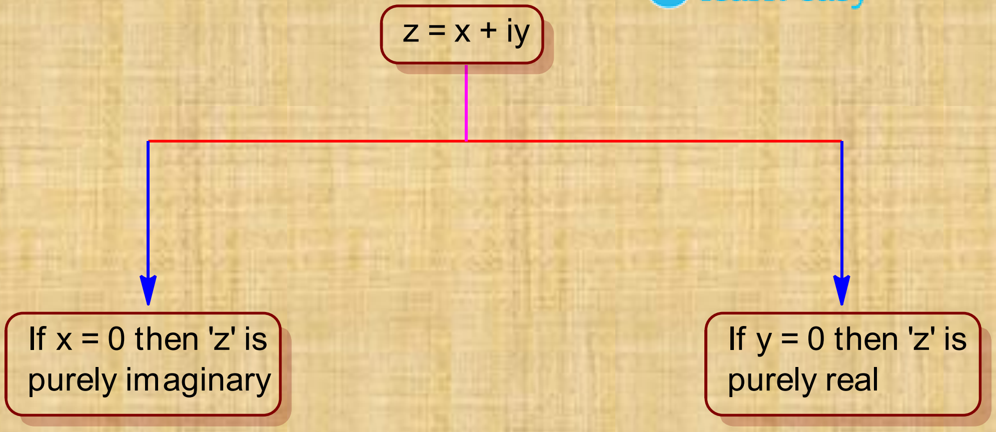
Thus, for z = 2 + 3i, Re (z) = 2 and $\text{Im}(z) = 3$.

For any complex number z, if Re(z) is zero then z is called as purely imaginary.

For **example**, z = 2i, -4i etc.

For any complex number z, if $\text{Im}(z)$ is zero then 'z' is called as purely real.

Thus,



Algebra of complex numbers:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ be any two complex numbers then,

1) $$z_1 \pm z_2 = (x_1 \pm iy_1) + (x_2 \pm iy_2) = (x_1 + x_2) + i(y \pm y_2)$$

2) $$z_1.z_2 = (x_1 + iy_1).(x_2 \pm iy_2) = x_1x_2 + ix_1y_2 + ix_2y_1 + i_2y_1y_2$$

$$= (x_1x_2 - y_1x_2) + i(x_1y_2 + x_2y_1)$$

3) $$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

$$1 = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \frac{(x_1x_2 + y_1y_2)}{x_2^2 + y_2^2} + \frac{i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

Atleast one of x_2 and y_2 is non – zero.

Additive inverse of z is –z.

For example, If z = 1 + i then additive inverse –z = -1 – i.

Multiplicative inverse of z is $\frac{1}{z}$ or z^{-1}.

i.e.. $z.z^{-1} = z^{-1}z = 1$

Let $z = x + iy \neq 0 \in C$ then there exist

$$z^{-1} = \frac{x}{x^2 + y^2} + i\left(\frac{-y}{x^2 + y^2}\right)$$

Such that $zz^{-1} = z^{-1}z = 1$

For example $z = 2 + 3i$ then,

$$z^{-1} = \left(\frac{2}{2^2 + 3^2}\right) + i\left(\frac{-3i}{2^2 + 3^2}\right)$$

$$z^{-1} = \frac{2}{13} + \frac{-3}{13}i$$