Chapter 5

COMPLEX NUMBERS

If $x + iy = \dfrac{1}{\cos\theta + i\sin\theta}$, show that $4x^2 - 1 = 0$.

Sol.

Given, $x + iy = \dfrac{1}{1 + \cos\theta + i\sin\theta}$

$$\left(\because\ 1+\cos A = 2\cos^2 A/2,\ \sin A = 2\sin A/2\cos A/2\right)$$

$$= \frac{1}{2\cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}$$

$$= \frac{1}{2\cos\theta/2 + 2\sin\theta/2\cos\theta/2}$$

$$= \frac{1}{2\cos\theta/2}\left[\cos\theta/2 - i\sin\theta/2\right]$$

$$= \frac{\cos\theta/2}{2\cos\theta/2} - \frac{i\sin\theta/2}{2\cos\theta/2}$$

$$= \frac{1}{2} - \frac{1}{2}i\tan\theta/2$$

Equating real part on both sides,

$$x = \frac{1}{2}$$

$$x^2 = \frac{1}{4} \Rightarrow 4x^2 - 1 = 0$$