Brainbox learn easy

Chapter 05

QUADRATIC EQUATIONS

Que: Show that none of the values of the function

$\frac{x^2+34x-71}{x^2+2x-7}$ **over R lies between 5 and 9**

Sol: Let $y=\frac{x^2+34x-71}{x^2+2x-7}$

$$\Rightarrow yx^2+2xy-7y=x^2+34x-71$$

$$\Rightarrow (y-1)x^2+(2y-34)x+(71-7y)=0$$

$$x\in R, b^2-4ac\geq 0 \; as \; (2y-34)^2-4(y-1)(71-7y)\geq 0$$

$$\Rightarrow 4y^2+1156-136y-4\left[71y-7y^2-71+7y\right]\geq 0$$

$$\Rightarrow 4y^2+1156-136y-284y+28y^2+284-28y\geq 0$$

$$\Rightarrow 32y^2-448y+1440\geq 0$$

$$\Rightarrow y^2-14y+45\geq 0$$

$$\Rightarrow (y-5)(y-9)\geq 0 \quad \begin{pmatrix} \because a(x-\alpha)(x-\beta)\geq 0 \\ x\in(-\infty,\alpha]\cup[\beta,\infty) \end{pmatrix}$$

$$\Rightarrow y\in(-\infty,5]\cup[9,\infty)$$

None of the values over R lies between 5 and 9