CHAPTER 2
RELATIONS

Equivalence relation:

A relation R on a set A is said to be a Equivalence relation on A. If

i) Reflective

ii) Symmetric

iii) Transitive

Note:

i) The least equivalence relation on a given set A is the identity relation on A.

ii) the greatest equivalence relation is universal relation.

E.g. Determine whether the following relation is equivalence relation.

Relation R in the set z of all integers defined as,

R = {(x, y) : x – y is an integer}

Sol.

Given relation R in the set z of all integers defined as,

R = {(x, y) : x – y is an integer} ------------ (i)

Is 'R' reflexive?

Let $x \in z$ put y = x in (i)

x – x = 0 is an integer which is true.

$\therefore$ $(x, x) \in R$ $\forall$ $x \in z$, R is reflexive relation.

Is R is symmetric?

Let $x \in z$, $y \in z$ and $(x, y) \in R$. B y (i). (x - y) is an integer.

$\therefore$ R is symmetric.

Is R is transitive?

Let $x \in y$, $y \in z$ and $z \in z$ such that $(x, y) \in R$ and $(y, z) \in R$.

By (i),

$x - y$ is an integer and $y - z$ is also integer.

Adding to eliminate y, $x - y + y - z$ = Integer + Integer

$x - z$ is an integer.

$\therefore$ R is transitive.

Hence, R is equivalence relation.

Inverse relation:

Let A and B be two sets and Let R be a relation from a set A to set B. Then the inverse of R, denoted by R^{-1}, is a relation B to A and is defined by

$R^{-1} = \{(b, a) : (a, b) \in R\}$

Also $\mathrm{dom}(R) = \mathrm{range}(R^{-1})$ and $\mathrm{range}(R) = \mathrm{dom}(R^{-1})$

E.g.

If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$

If $R = \{(1, a), (2, a), (3, b), (3, c)\}$

$R^{-1} = \{(a, 1), (a, 2), (b, 3), (c, 3)\}$