Chapter 8

APPLICATION OF INTEGRALS

Problem 1:

Find the area enclosed between the curves $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.

Sol.

$$\text{Required area} = \int_{-1}^{2}\left(x^3+3\right)dx$$

$$= \frac{x4}{4}+3x|_{-1}^{2}$$

$$= \left\{\frac{2^4}{4}+3(2)\right\}-\left\{\frac{(-1)^4}{4}+3(-1)\right\}$$

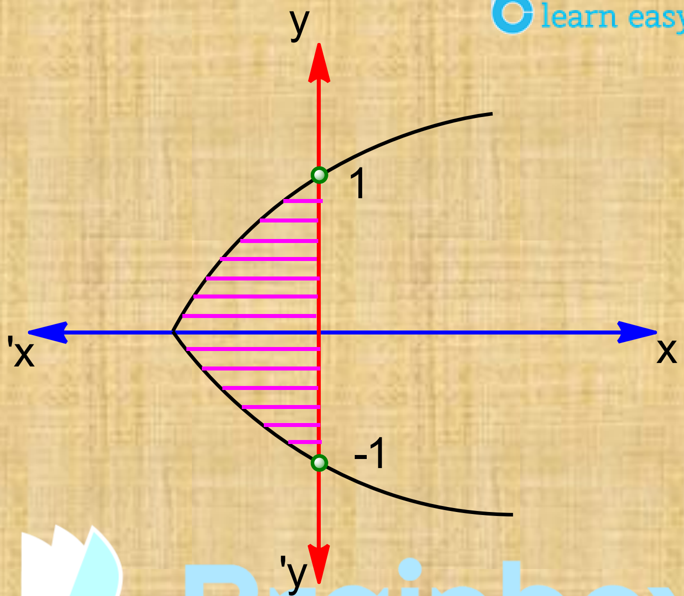
$$= (4+6)-\left(\frac{1}{4}-3\right) \text{ After simplification}$$

$$= \frac{51}{4} \text{ Sq. Unit}$$

Problem 2:

Find the area cube between x = 0, $2x = y^2 - 1$.

Sol.



$x = 0,\ 2x = y^2 - 1 \Rightarrow y - 1 = 0 \Rightarrow y = \pm 1$

Required area $= -\int_{-1}^{1} x\,dy = -\int_{-1}^{1}\left(\frac{y^2-1}{2}\right)dy$

$= -2\int_{0}^{1}\left(\frac{y^2-1}{2}\right)dy \qquad \because \frac{y^2-1}{2}$ - Sum function

$= -\frac{\cancel{2}}{\cancel{2}}\left\{\frac{y^3}{3} - y\right\}_0^1$

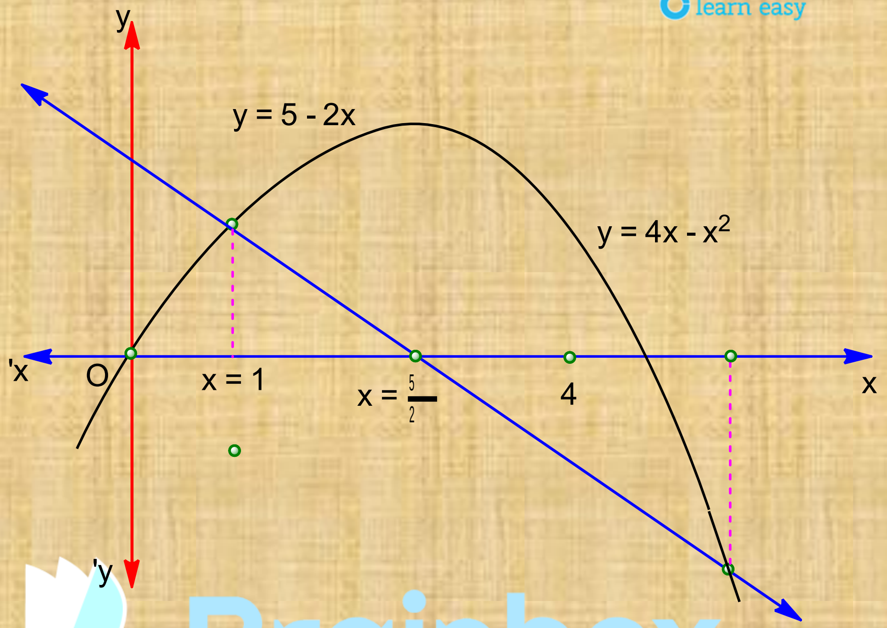
$= -\left\{\left(\frac{1}{3} - 1\right) - (0)\right\}$

$= -\frac{1}{3} + 1 = \frac{2}{3}$ **Sq. Units**

Problem 3:

Find the area enclosed between the curves $y = 4x - x^2$, $y = 5 - 2x$.

Sol.



Given curves are $y = 4x - x^2$ ----------- (1)

$y = 5 - 2x$ ------------- (2)

Solving (1) & (2), we get points of intersection,

$4x - x^2 = 5 - 2x \Rightarrow x^2 - 6x + 5 = 0$

$\Rightarrow (x - 1)(x - 5) = 0$

$X = 1, 5.$

The required region lies between the given curves from $x = 1$ to $x = 5$.

Required area = $\int_1^5 \left[\left(4x - x^2\right) - \left(5 - 2x\right)\right] dx$

$A = \int_1^5 \left[\left(6x - x^2 - 5\right)\right] dx$

$$= 6.\frac{x^2}{2} - \frac{x^2}{3} - 5x\Big|_1^5 = \left\{\frac{6.(5)^2}{2} - \frac{5^3}{3} - 5(5)\right\} - \left\{\frac{6.(1)^2}{2} - \frac{1^3}{3} - 5(1)\right\}$$

After simplification will get,

$$\frac{25}{3} + \frac{7}{3} = \frac{32}{3}$$ **Sq. Unit**