Chapter 8

APPLICATION OF INTEGRALS

Problem 4:

Find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

Sol.

Given,

$y^2 = 4ax$ ---------- (1)

$x^2 = 4by$ ---------- (2)

From (2), $y = \frac{x^2}{4b}$ ---------- (3)

$\left(\frac{x^2}{4b}\right)^2 = 4ax$ $\because$ From (3)

$\frac{x^4}{16b^2} = 4ax$

$\frac{x^4}{16b^2} - 4ax = 0$

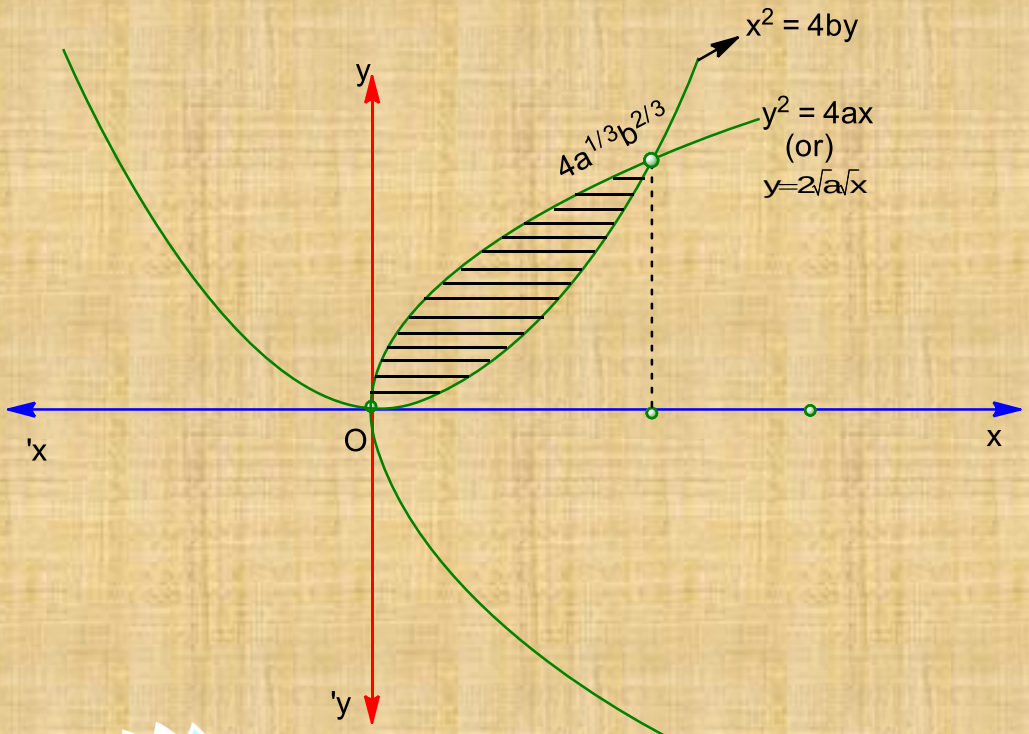
$x\left(\frac{x^3}{16b^2} - 4a\right) = 0$

X = 0 (or) $x^3 = 16b^2 \times 4a$.

X = 0 (or) $x^3 = 4^3ab^2$

X = 0 (or) $x = 4a^{1/3} b^{2/3}$

Required area = $\int_0^{4a^{1/3}b^{2/3}} (y_{UC} - y_{LC})dx$



$$= \int_0^{4a^{1/3}b^{2/3}} \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4b}\right) dx$$

$$= 2\sqrt{a} \int_0^{4a^{1/3}b^{2/3}} \sqrt{x}\,dx - \frac{1}{4b} \int_0^{4a^{1/3}b^{2/3}} x^2 dx$$

$$= 2\sqrt{a}\frac{2}{3}.x^{3/2} - \frac{x^3}{12b}\Bigg|_0^{4a^{1/3}b^{2/3}}$$

$$= \frac{4}{3}\sqrt{a}\left(4a^{1/3}b^{2/3}\right)^{3/2} - \frac{1}{12b}\left(4a^{1/3}b^{2/3}\right)^3$$

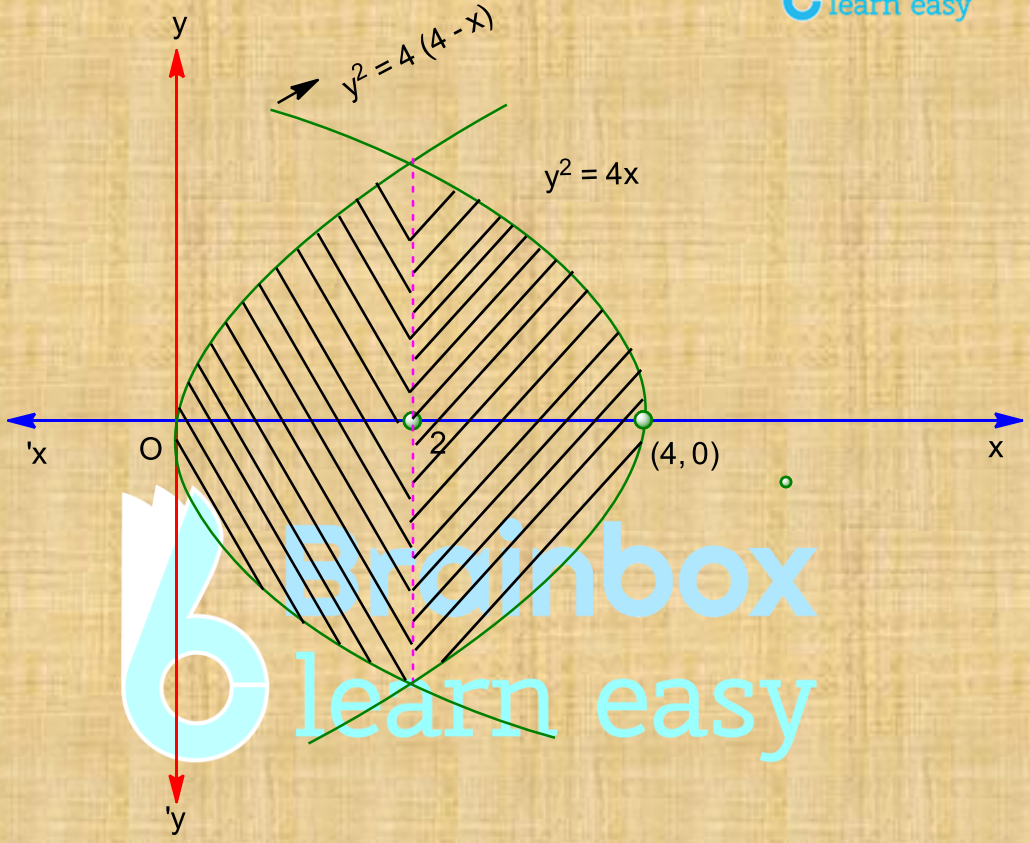
After Simplification,

$$= \frac{32}{3}ab - \frac{16}{3}ab = \frac{16ab}{3}$$ **Sq. Units**

Problem 5:

Find the area enclosed between $y^2 - 4x$, $y^2 = 4(4 - x)$.

Sol.



Given curves, $y^2 = 4x$ ---------- (1)

$y^2 = 4(4 - x)$ ---------- (2)

Solving (1) & (2), we get

$4x = 4(4 - x) = 16 - 4x$

$\Rightarrow 8x = 16$

$\Rightarrow x = 2$

Required area $= 2\left[\int_0^2 \sqrt{4x}\,dx + \int_0^4 \sqrt{4(4-x)}\,dx\right]$

After Simplification,

$$= 2\left[\left\{2x\frac{2}{3}2x^{3/2}\right\}_0^2 + \left(2x\frac{2}{3}(4-x)^{3/2}(-1)\right)\right]_2^4$$

$$= \frac{8}{3}\left[2^{3/2} - 0 + (4-2)^{3/2}\right]$$

$$= \frac{8}{3} \times 2 \times 2^{3/2} = \frac{32\sqrt{2}}{3} \text{ Sq. Units.}$$