

Chapter 13

PROBABILITY

BAYE'S THEOREM:

Suppose $A_1, A_2, \dots, A_n$ are n mutually exclusive and exhaustive events of a random experiment associated with sample space S such that $P(A_i) \neq 0$, and A is any event which takes place in conjunction with any one A_i .

$$P(A_k/A) = \frac{P(A_k)P(A/A_k)}{\sum_{i=1}^n P(A_i)P(A/A_i)}, \text{ for any } k=1, 2, \dots, n.$$

Proof:

Given that $P(A_i) \neq 0$ for $i=1, 2, 3, \dots, n$

Since, $A_1, A_2, A_3, \dots, A_n$ are mutually exclusive.

$A_i \cap A_j = \emptyset$ for $i \neq j$ and $1 \leq i, j \leq n$

Since, $A_1, A_2, \dots, A_n$ are exhaustive $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n = S$

Here for any event A of S we have $(A \cap A_i), (A \cap A_j)$ are also exclusive for $i \neq j$.

$$\begin{aligned} P(A) &= P(A \cap S) \\ &= P[A \cap (A_1 \cup A_2 \cup \dots \cup A_n)] \\ &= P[(A \cap A_1) \cup (A \cap A_2) \cup (A \cap A_3) \dots \cup (A \cap A_n)] \\ &= P(A \cap A_1) + P(A \cap A_2) + P(A \cap A_3) \dots + P(A \cap A_n) \\ &\quad \text{(By axiom of additivity)} \\ &= P(A_1)P(A/A_1) + P(A_2)P(A/A_2) + \dots + P(A_n)P(A/A_n) \end{aligned}$$

$$= \sum_{i=1}^n P(A_i)P(A/A_i) \rightarrow (1)$$

Consider for $1 \leq k \leq n$

$$P(A_k/A) = \frac{P(A_k \cap A)}{P(A)}$$

(By definition of conditional probability)

$$\begin{aligned} &= \frac{P(A_k)P(A/A_k)}{P(A)} \\ &= \frac{P(A_k)P(A/A_k)}{\sum_{i=1}^n P(A_i)P(A/A_i)} \quad (\text{From eq(1)}). \end{aligned}$$

