

Chapter 13

PROBABILITY

Example:

Three urns have the following composition of balls.

Urn-I: 1 white, 2 black

Urn-II: 2 white, 1 black

Urn-III: 2 white, 2 black

One of the urns is selected at random and a ball is drawn.

It turns out to be white. Find the probability that it come from urn-III.

Solution:

Let A_1, A_2, A_3 be the events of selecting urn-I, urn-II, urn-III respectively.

$$\text{Then } P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$$

Now A_1, A_2, A_3 are mutually exclusive and exhaustive events.

Let E be the event of drawing a white ball from the selected urn.

$$P(E/A_1) = \frac{1}{3}, \quad P(E/A_2) = \frac{2}{3}, \quad P(E/A_3) = \frac{2}{4} = \frac{1}{2}$$

$$P(A_3/E) = \frac{P(A_3) \cdot P(E/A_3)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{1}{6}}{\frac{1}{9} + \frac{2}{9} + \frac{1}{6}} = \frac{\frac{1}{6}}{\frac{2+4+3}{18}} = \frac{3}{9} = \frac{1}{3}$$