

Chapter 13

PROBABILITY

ADDITION THEOREM ON PROBABILITY:

If A, B are any two events of a random experiment and P is a probability function then,

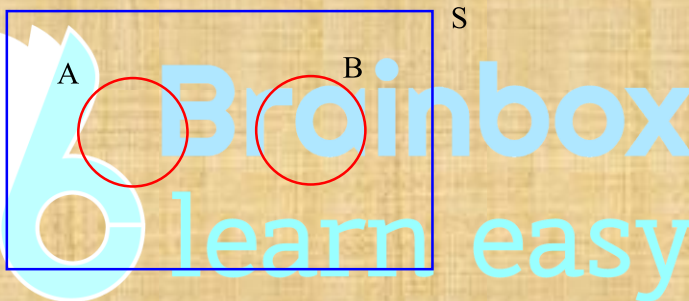
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Proof:

Case(i):

Suppose A and B are mutually exclusive.

i.e., $A \cap B = \phi$ from Axiom of additivity.

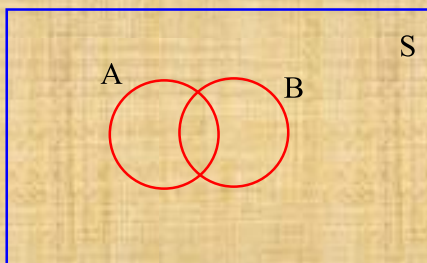


$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= P(A) + P(B) - 0 \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

Case(ii):

Suppose A and B are not mutually exclusive

i.e., $A \cap B \neq \phi$, there $A \cup B = A \cup (B - A)$



Therefore,

$$\begin{aligned}
 P(A \cup B) &= P[A \cup (B - A)] \\
 &= P(A) + P(B - A) \\
 &= P(A) + P[B - (A \cap B)] \\
 &= P(A) + P(B) - P(A \cap B) \quad \because A \cap B \subseteq B \\
 \therefore P(A \cup B) &= P(A) + P(B) - P(A \cap B)
 \end{aligned}$$

NOTE:

- i. If A and B are mutually exclusive events, then
 $A \cap B = \phi$ and hence $P(A \cap B) = 0$.
 $\therefore P(A \cup B) = P(A) + P(B)$
- ii. Two events A and B are mutually exclusive if and only, If $P(A \cup B) = P(A) + P(B)$.

$$\text{iii. } 1 = P(S) = P(A \cup A^1) = P(A) + P(A^1) \quad \because A \cap A^1 = \phi$$

$$\text{Or } P(A^1) = 1 - P(A)$$

If A, B and C are any three events in a sample spaces, then

$$\begin{aligned}
 P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \\
 &\quad - P(C \cap A) + P(A \cap B \cap C)
 \end{aligned}$$

- iv. If A, B, C are mutually exclusive events, then

$$A \cap B = \phi, B \cap C = \phi, C \cap A = \phi, A \cap B \cap C = \phi$$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C).$$

- v. If A and B are any two events, then

$$(A - B) \cap (A \cap B) = \phi \text{ and } A = (A - B) \cup (A \cap B)$$

$$\therefore P(A) = P(A - B) + P(A \cap B)$$

$$P(A) = P(A \cap B^1) + P(A \cap B) \text{ or}$$

$$P(A) - P(A \cap B) = P(A \cap B^1)$$

$$\text{Similarly, } P(B) - P(A \cap B) = P(B \cap A^1).$$

Example-1:

Ram and Vijay are two contestants for an election,

probability that Ram will win is $\frac{1}{4}$ and that Vijay will win

the election is $\frac{1}{5}$, then what is probability that either Ram or Vijay will win the Election.

Solution:

From data probability that Ram will win the Election is

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{5}$$

$P(A \cap B) = 0$, Since both of them cannot win together.

$$\text{So } P(A \cup B) = P(A) + P(B) = \frac{1}{4} + \frac{1}{5} = \frac{2}{20} = \frac{1}{10}.$$

Example- 2:

A, B, C are three horses in a race. The probability of A to win the race is twice that of B, and the probability of B is twice that of C. What are the probabilities of A, B and C to win the race ?

Solution:

Let A, B, C be the events that the horses A, B, C win the race respectively.

$$\text{Given } P(A) = 2P(B), P(B) = 2P(C)$$

$$\therefore P(A) = 2P(B) = 2[2P(C)] = 4P(C)$$

Since the horses A, B and C run the race, $A \cup B \cup C = S$ and A, B, C are mutually disjoint.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$\Rightarrow P(S) = 4P(C) + 2P(C) + P(C)$$

$$1 = 7P(C) \Rightarrow P(C) = \frac{1}{7}$$

$$P(A) = 4P(C) = \frac{4}{7}, \quad P(B) = 2P(C) = \frac{2}{7}.$$

