

Chapter 13

PROBABILITY

MULTIPLICATION THEOREM OF PROBABILITY:

Let A and B be two events associated with a sample space S.

Clearly, the set $A \cap B$ denotes the event that both A and B have occurred. In other words, $A \cap B$ denotes the simultaneous occurrence of the events A and B.

The event $A \cap B$ is also written as AB.

For example, In the experiment of drawing two cards one after the other, we may be interested in finding the probability of the event 'a spade and a diamond'.

The probability of event AB is obtained by using the conditional probability.

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}; P(B) \neq 0$$

$$\therefore P(A \cap B) = P(A/B)P(B) \rightarrow (1)$$

$$\text{Also, } P(B/A) = \frac{n(B \cap A)}{n(A)}; P(A) \neq 0$$

$$\therefore P(A \cap B) = P(B/A)P(A) \rightarrow (2)$$

From (1) & (2), we also get

$$P(A \cap B) = P(A/B)P(B) = P(B/A)P(A), \text{ if } P(A), P(B) \neq 0$$

This result is known as multiplication rule of probability.

More than two Events:

If A, B and C are three events of sample space, we have

$$\begin{aligned}
 P(A \cap B \cap C) &= P(A)P(B/A)P(C/A \cap B) \\
 &= P(A)P(B/A)P(C/AB)
 \end{aligned}$$

Example:

An Urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that first ball is black and second ball is white?

Solution:

Let A and B denote, respectively, the events that first ball drawn is black and second ball drawn is white. We have to find $P(A \cap B)$ or $P(AB)$

$$P(A \cap B) = P(A)P(B/A)$$

Now, $P(A) = P(\text{black ball in first draw}) = \frac{10}{15} = \frac{2}{3}$.

Also, given that the first ball drawn is black, i.e., Event A has occurred. Now, there are 9 black balls and 5 white balls left in the urn.

$\therefore$ The probability that the second ball drawn is white, given that the ball in the first drawn is black, is nothing but the conditional probability of B given that A is occurred.

$$\therefore P(B/A) = \frac{5}{14}$$

$$\therefore P(A \cap B) = P(A)P(B/A) = \frac{2}{3} \times \frac{5}{14} = \frac{5}{21}.$$